Component separation in Herschel maps based on the coherence analysis with atomic and molecular gas

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We present the first results of a new component separation technique of the galactic emission based on the coherence analysis of anisotropic complex wavelets coefficients. The analysis compares common structures at multiple scales between maps instead of their pixel intensity distribution like classical correlation analysis. The technique is applied at 160, 250, 350 and 500 μm on a ~ 1° x 1° field of the galactic plane observed within the Hi-GAL survey of the Herschel space telescope. The galactic emission is separated in four gaseous components, the ^{12}CO column density map, the HI line velocity channel showing the most contrasted HI self-absorption features, the HI column density map of the galactic external arm and the residue map showing the emission not traced by the other components. All coherence spectra show a peak at ~0.04 arcmin^{-1} which could be the characteristic scale where the dust emission becomes significantly coherent with gas components. Spectral energy distributions are fitted to all separated components. In all regions, the emissivity spectral index β is significantly lower for the original map and the dust emission coherent with the ^{12}CO comes from the colder dust. In one subregion, HISA and HI components show similar dust temperatures and β values. The lower β value for the original map and the other HISA component could be explained by the presence of more than one gas component with different temperatures along the line of sight.

Introduction

To help understand physical mechanisms responsible for the phase transition from atomic to molecular gas, and more generally the formation of molecular clouds, studying the interstellar cold dust emission is imperative for its role as a catalyst in H_2 formation. It is also, unfortunately in our case, the best tracer of the total amount of matter along any galactic line of sights. Hence, separating the contribution of the infrared emission associated with the atomic gas from the one with the molecular component at low galactic latitude is necessary. We present a correlation technique based on the coherence analysis of spatial scales decomposed with complex wavelet transforms. Contrary to classical correlation analysis, this method is based on the identification of common structures between maps of dust, HI and CO emission. This more precise component separation technique applied to multiple wavelength Herschel maps allows the characterization of cold dust properties as a function of the gas phase in the confused galactic plane.

Coherence analysis

Coherence is a measure of the phase relationships between two signals. The technique is very similar to the classic correlation analysis except that the comparison between variables is made in the frequency domain. It can be defined as

$$\gamma^2_{fg}(\mathbf{k}) = \frac{\overbrace{|\langle \hat{f}^*(\mathbf{k})\hat{g}(\mathbf{k})\rangle|^2}^{\text{cross-spectrum}}}{\langle \hat{f}^*(\mathbf{k})\hat{f}(\mathbf{k})\rangle \underbrace{\langle \hat{g}^*(\mathbf{k})\hat{g}(\mathbf{k})\rangle}_{\text{power spectrum}}},$$

where $\hat{f}(\mathbf{k})$ and $\hat{g}(\mathbf{k})$ are the Fourier transforms of $f(\mathbf{x})$ and $g(\mathbf{x})$. If the phases of the cross-spectra are randomly distributed, the averaging cancels the coherence which signifies that the two signals are uncorrelated. If the phases of the cross-spectra interfere constructively, then the averaging operation produces a high coherence.

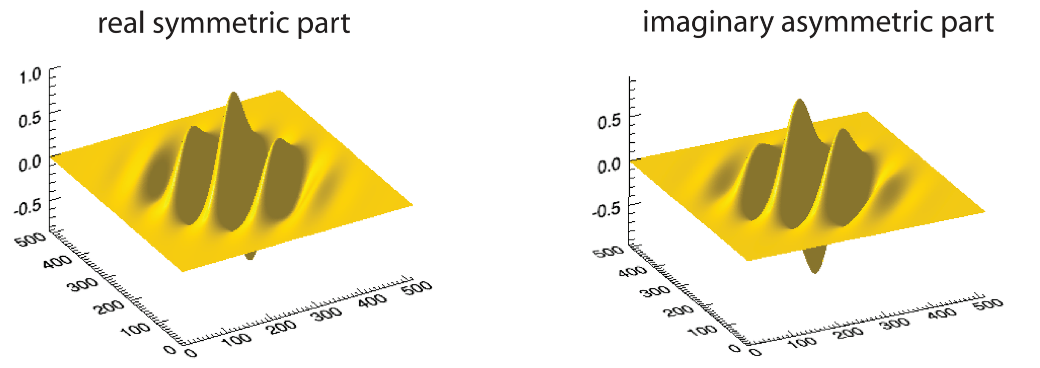
In this project, we take advantage of the anisotropy, local property and phase information provided by the complex Morlet wavelet to calculate this function. The coherence spectrum using wavelet functions becomes,

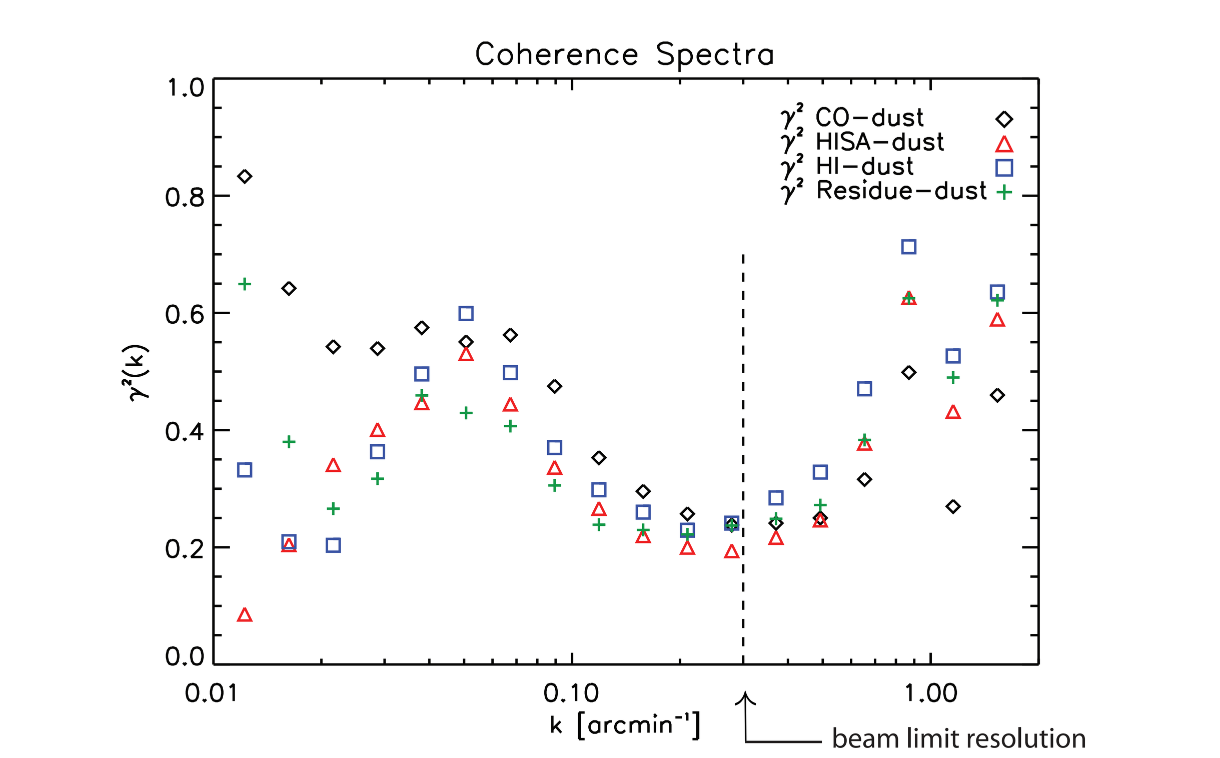
$$\gamma^2_{fg}(\underset{\text{scale}}{l}, \mathbf{x}) = \frac{|\langle \tilde{f}^*(l,\theta,\mathbf{x})\tilde{g}(l,\theta,\mathbf{x})\rangle|^2}{P_f(l,\mathbf{x})P_g(l,\mathbf{x})}.$$

The average operation, $\langle\rangle$, can be made over azimuthal angles θ or the position $\mathbf{x}$. The properties the Morlet wavelet transform are very similar to the Fourier transform. The wavelet itself is simply sinusoidal functions multiplied by a standard normal distribution,

$$\begin{aligned}\psi(\mathbf{x}) &= e^{i\mathbf{k_0}\mathbf{x}}e^{-|\mathbf{x}|^2/2} \\ &= [\cos(\mathbf{k_0}\mathbf{x}) - i\sin(\mathbf{k_0}\mathbf{x})]e^{-|\mathbf{x}|^2/2}.\end{aligned}$$

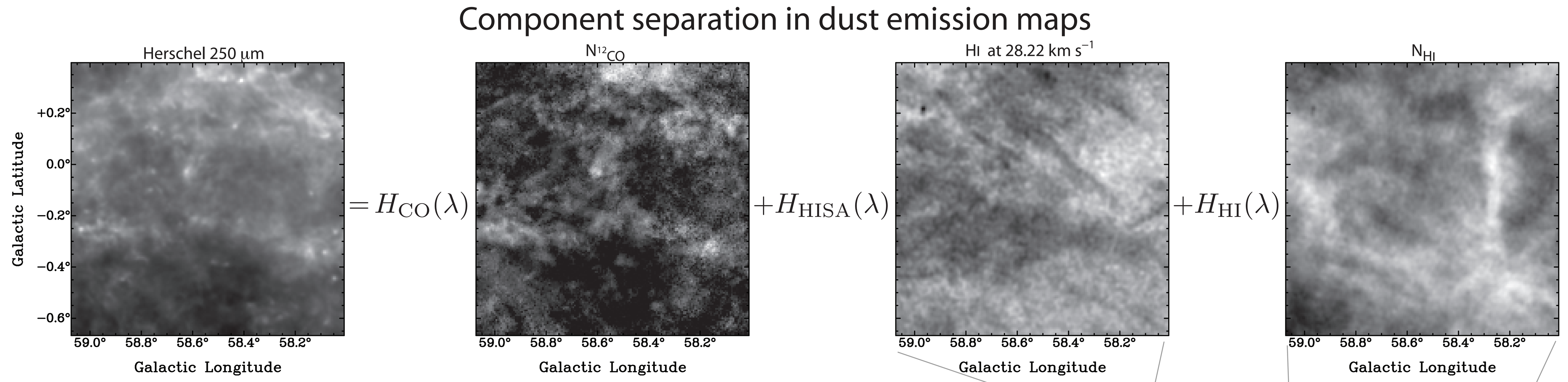
Anisotropic Morlet Wavelet

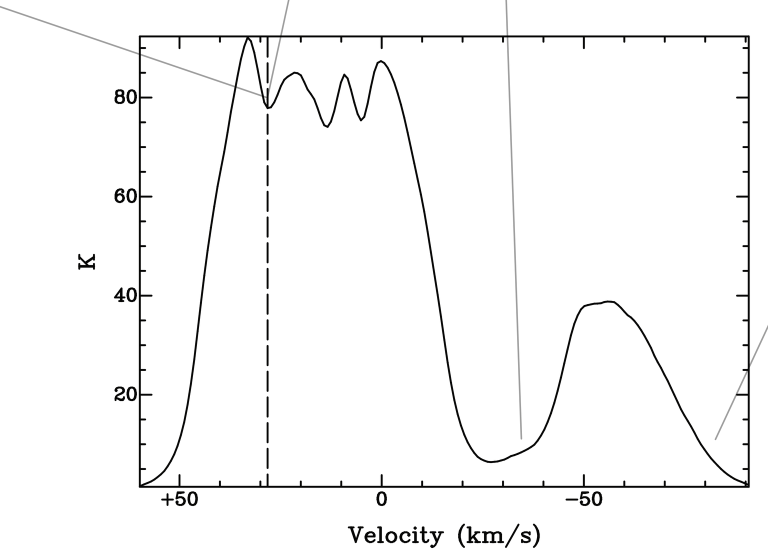




The coherence is slightly superior in general between the ^{12}CO map and the dust emission. All coherence spectra show a peak at ~0.04 arcmin^{-1}. In the case of γ^2 CO–dust, coherence coefficients are higher at coarser scales. The spectra of γ^2 HISA–dust and γ^2 HI–dust are similar. The lower coefficients for the HISA velocity channel can be caused by the presence of both HI emission and absorption in the same map. The residue coherence coefficients show similar characteristics to the three other spectra.

Component separation in dust emission maps





This linear model is usually solved using the inversion method which is based on the calculation of the covariance matrix of all elements. One important limitation of the inversion method is that only the mean emissivity of the dust component can be determined for a given map. The method can not give the spatial variation of dust properties such as the emissivity spectral index or the temperature.

We propose here a similar model but solved in the complex wavelet space. As for the coherence analysis (see the left box), the calculation is based on the phase relationships between structures present in every map without the derivation of the covariance matrix.

Example of a two components model :

Complex wavelet coefficient of an output signal; Linear frequency response function; Complex wavelet coefficient of an input signal; Noncoherent input signal

$$\tilde{y}(l,\mathbf{x}) = \underbrace{H(l,\mathbf{x})\cdot\tilde{x}(l,\mathbf{x})}_{\text{Coherent input signal}} + \tilde{N}(l,\mathbf{x})$$

Cross spectrum; Vanishes because noncoherent

$$\tilde{x}^*\tilde{y} = H\cdot\tilde{x}^*\tilde{x} + \cancel{\tilde{x}^*\tilde{N}}$$

Definition of the linear frequency response function

$$\boxed{H(l,\mathbf{x}) = \frac{\langle\tilde{x}^*(l,\mathbf{x})\tilde{y}(l,\mathbf{x})\rangle_\theta}{\langle|\tilde{x}(l,\mathbf{x})|^2\rangle_\theta}}$$

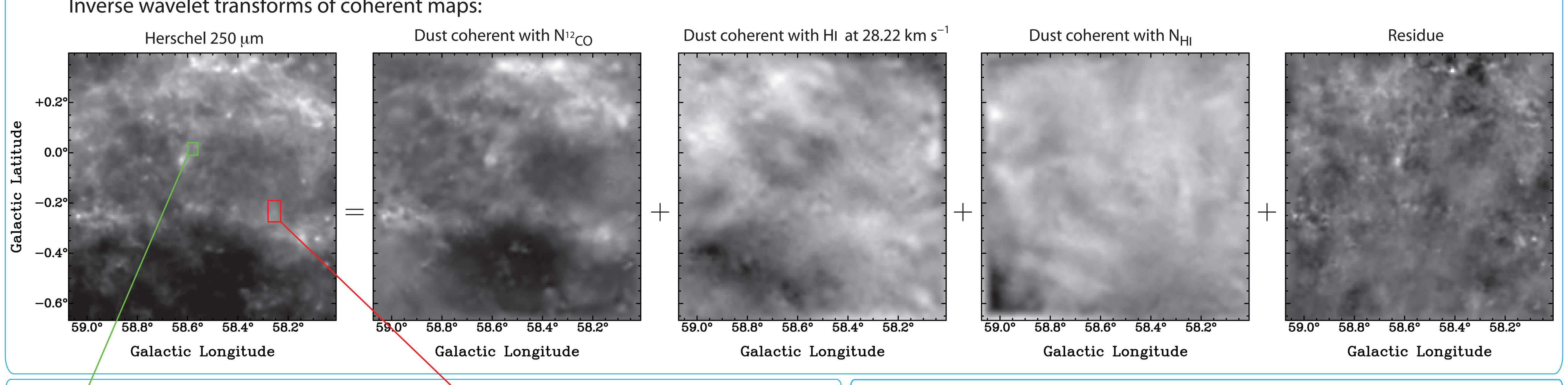
Comparison with the correlation relationship between two sets of data

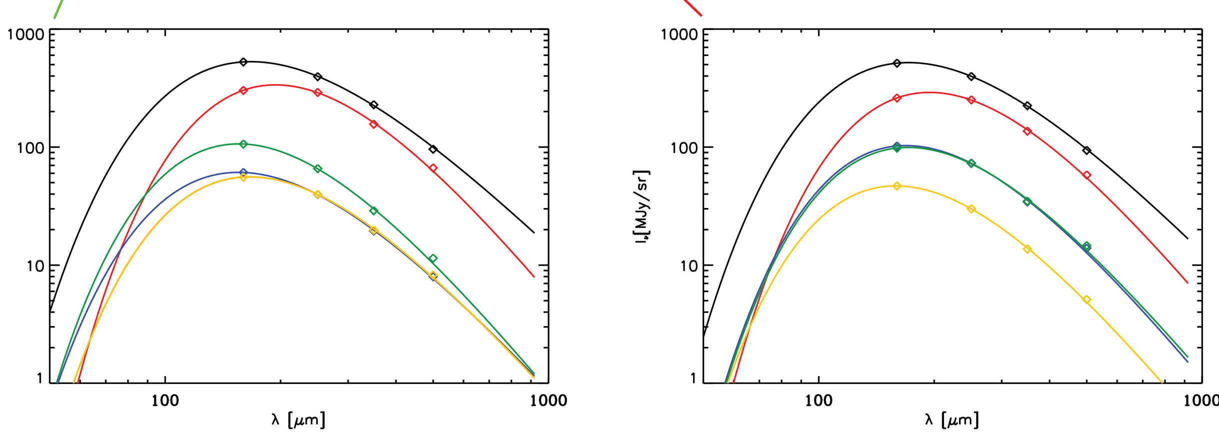
$$\rho_{g:f} = \frac{\sigma_{fg} \rightarrow \text{Covariance}}{\sigma_f^2 \rightarrow \text{Variance}}$$

The linear frequency response function $H(l,\mathbf{x})$ is similar to the correlation relation in the spatial domain but is not equal to its Fourier transform. The linear frequency response is a far more powerful tool than the classical correlation relationship between two data because of its scale and coordinate dependency.

A goal of this project is to specify the role of the cold HI phase in the formation of molecular gas in the ISM. This specific region has been choosen in the Hi-GAL survey for the abundance of HI self absorption (HISA) detected along the line of sight. The HI emission line is very complicated to interpret in the galactic plane because of the mixture of two phases, the warm and the cold neutral medium. For this reason the real amount of emitted photons that are absorbed through a long line of sight in the galactic plane is practically impossible to establish accurately. Several HI regions seen in emission at a given velocity is probably a blend of warm and cold neutral gas which can hardly be dissociated from one another. However, for particular geometries, some colder gas structures can appear distinctively in absorption against a warmer HI background. The exact amount of gas present in those HISA structures is again very hard to determine accurately in part because of the unknown fraction of warmer gas emitting in the foreground of the absorbing structure. In spite of these unknown variables, HISAs can provide very precious information when one uses a coherence analysis based on structures present in the map instead of its pixel intensity distribution.

Inverse wavelet transforms of coherent maps:





	T [K]	β
◇ Total	19.82	1.39
◇ ^{12}CO	13.86	2.37
◇ HISA	19.51	1.78
◇ HI	17.61	2.28
◇ Residue	17.59	1.96

	T [K]	β
◇ Total	18.58	1.57
◇ ^{12}CO	13.88	2.35
◇ HISA	16.08	2.33
◇ HI	16.15	2.24
◇ Residue	17.21	2.29

The separation technique is applied on PACS 160 μm and SPIRE 250, 350, 500 μm maps. The spectral energy distribution (SED) is fitted to all separated components shown above. In both regions, the β value is significantly lower for the original map and the dust emission coherent with the ^{12}CO comes from the colder dust. The HISA and HI components show similar dust temperatures and β values for the lower region. The residue on this location could trace warmer HI moving at positive velocity according to the HI profile shown in the box above. In the upper region, the HISA component can trace a mix of absorbed and emitted HI line at different temperatures which could have an effect on the β value. The same mixing effect could also explain small β values for the original map.

Conclusion

As far as we know, we realized the first 2-D coherence analysis using an anisotropic complex wavelet and its reconstruction formula. Applied on four gas components, the technique successfuly traces common structures seen in emission, in the case of column density maps of ^{12}CO and HI, as well as features seen in absorption, in the case of a HI velocity channel. Using this separation technique at several wavelength, SEDs have been fitted on two subregions of each separated gas component. All fits show reliable temperature and β index values. In both subregions, dust emission associated with ^{12}CO structures are colder than dust emission associated with HI or HISA features. Lower values of β could indicate a mixing effect of gas components at different temperatures along the line of sight. With those preliminary calculations, we show that this new separation technique can provide the spatial variation of dust properties as a function of gas components.

Coherence spectra can also be measured for each separated gas component. For this field, a common peak at ~0.04 arcmin^{-1} suggests that gas components begin to be coherent with the dust emission at this characteristic scale.

In future studies, this new separation technique will be tested on other regions of the galactic plane where HISA features are present. Characterizing properties of dust emission coherent with HISA features will help to understand physical conditions driving cooling processes of HI gas. Ultimately, this analysis could define more precisely physical mechanisms responsible for the phase transition from atomic to molecular gas.