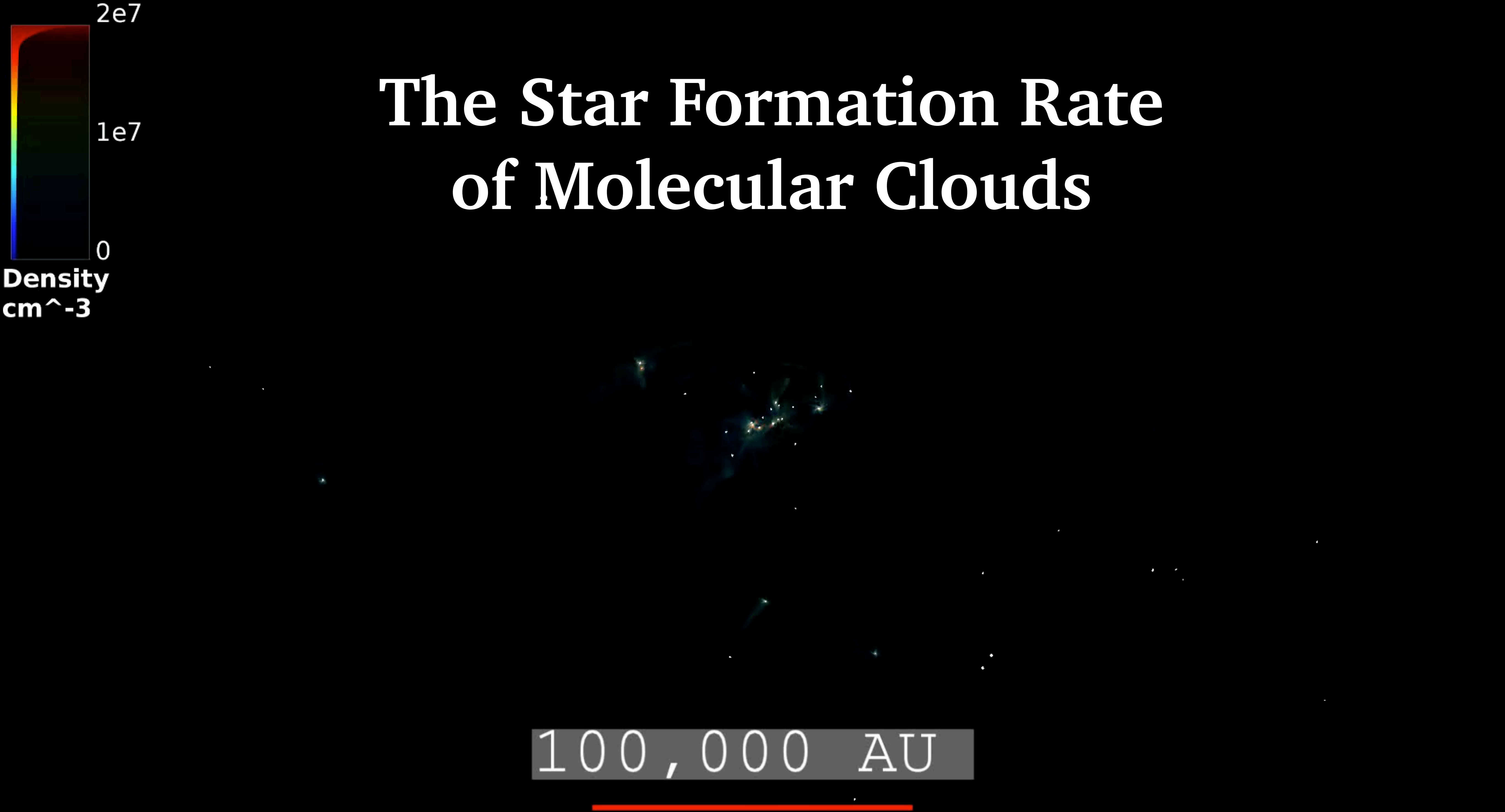
The Star Formation Rate of Molecular Clouds



Paolo Padoan
(ICREA - Institute of Cosmos Sciences - University of Barcelona)

Plan of the talk

- The SFR problem
- Analytical approach
- Numerical modeling
- Application to galaxies

Strong bias toward the turbulent fragmentation scenario

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We want to model the SFR to understand the formation and evolution of galaxies. So we study the fragmentation of large ISM regions.

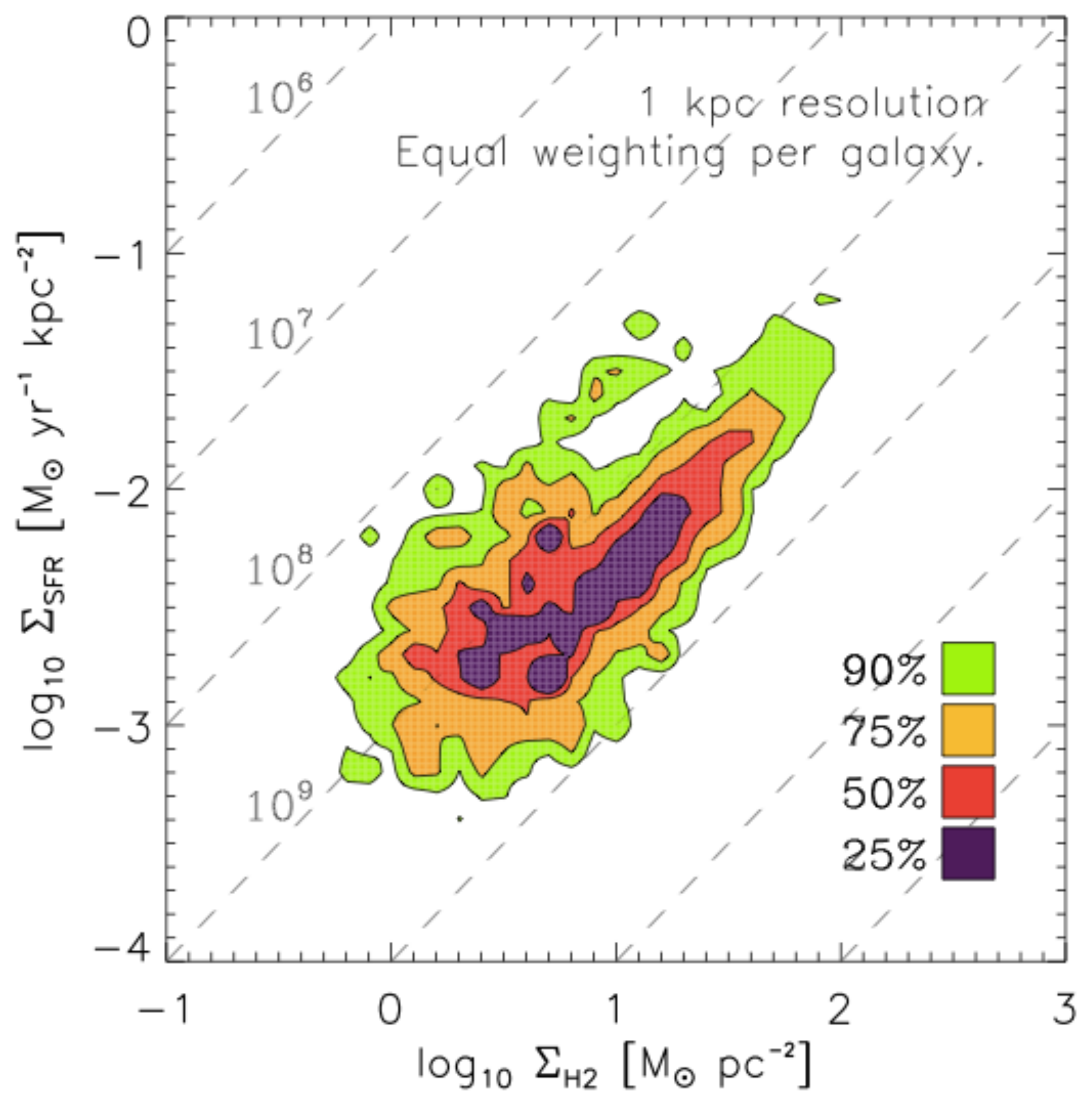
We may refer to them as **‘molecular-clouds’**, in the context of disk galaxies, but we really mean any region of the cold ISM between 1 and 100 pc.

Could even be much larger scale at high-z (e.g. a big chunk of a protogalaxy), when the turbulence is driven by mergers or cold accretion.

So the approach is general; ‘molecular clouds’ are just ideal sites to test the theory.

Disk galaxies consume their molecular gas in ~1 Gyr.
On all scales: **SFR ≈ 0.02** M/t_{ff}

Why is gravity so inefficient?

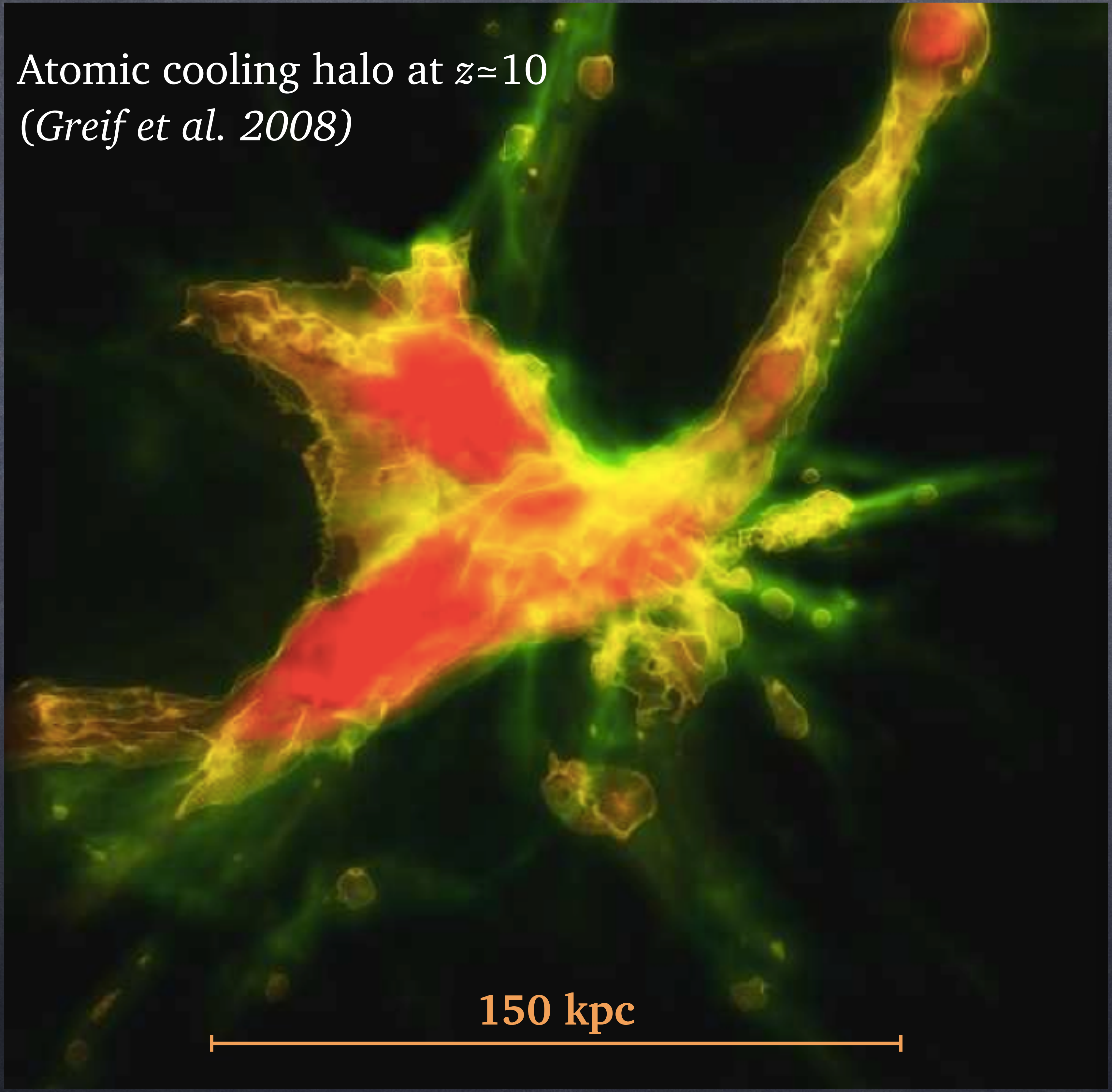


Bigiel et al. (2011)

Because the ISM is turbulent.

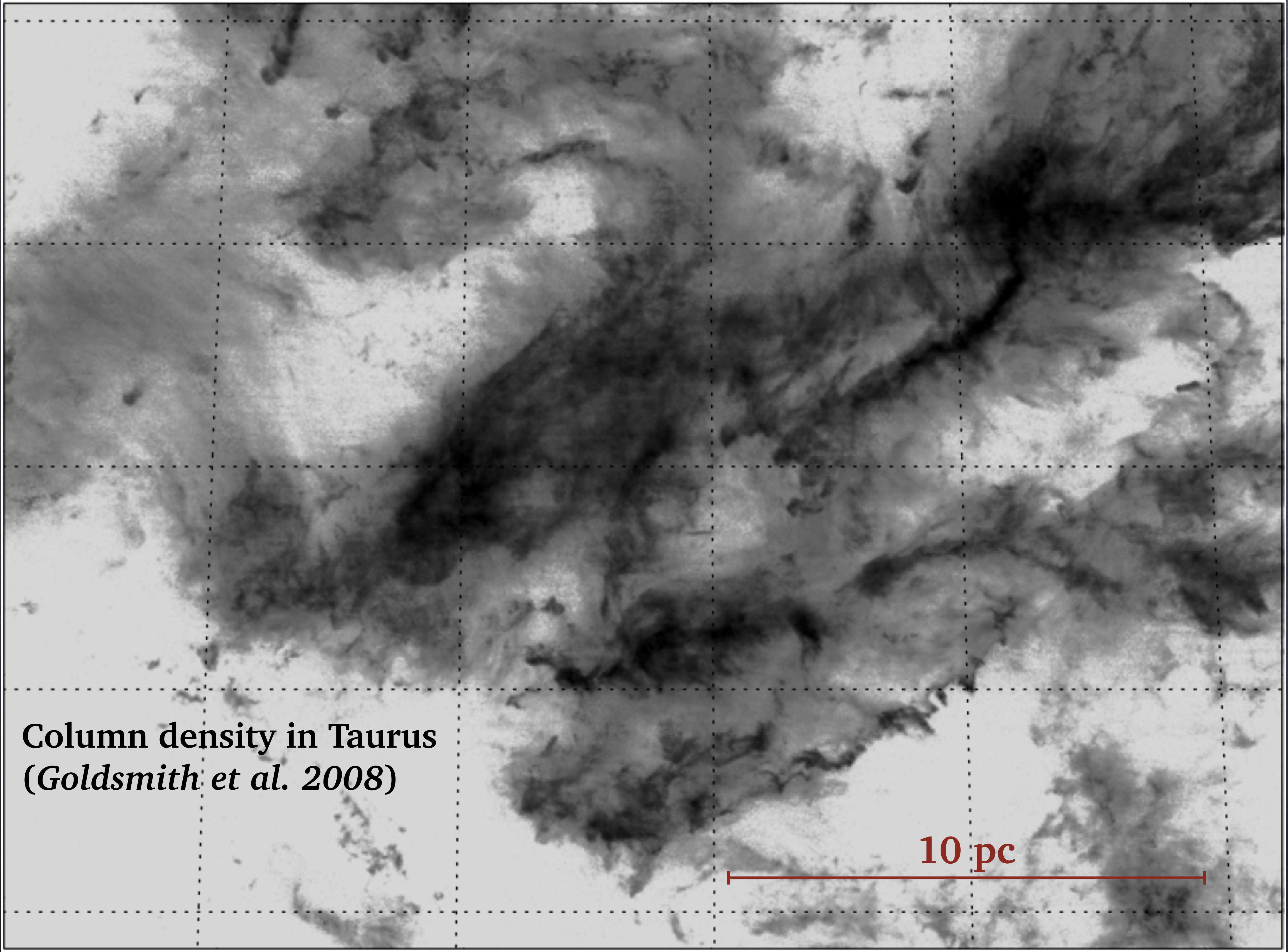
First Galaxies

Supersonic turbulence from cold accretion and streaming velocities



Giant Molecular Clouds

Supersonic turbulence from SN explosions

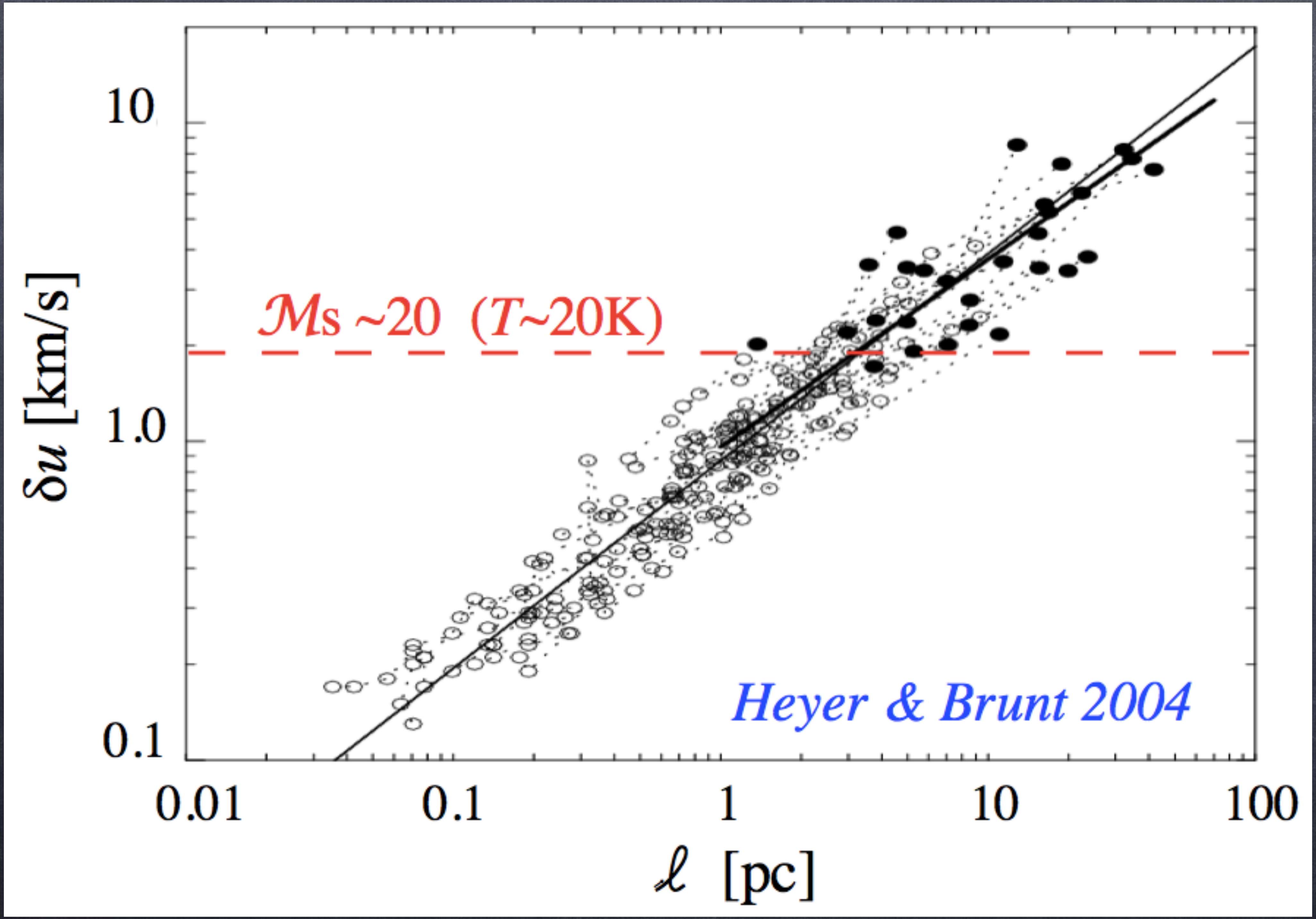


Column density in Taurus
(*Goldsmith et al. 2008*)

Supersonic Turbulence in GMCs

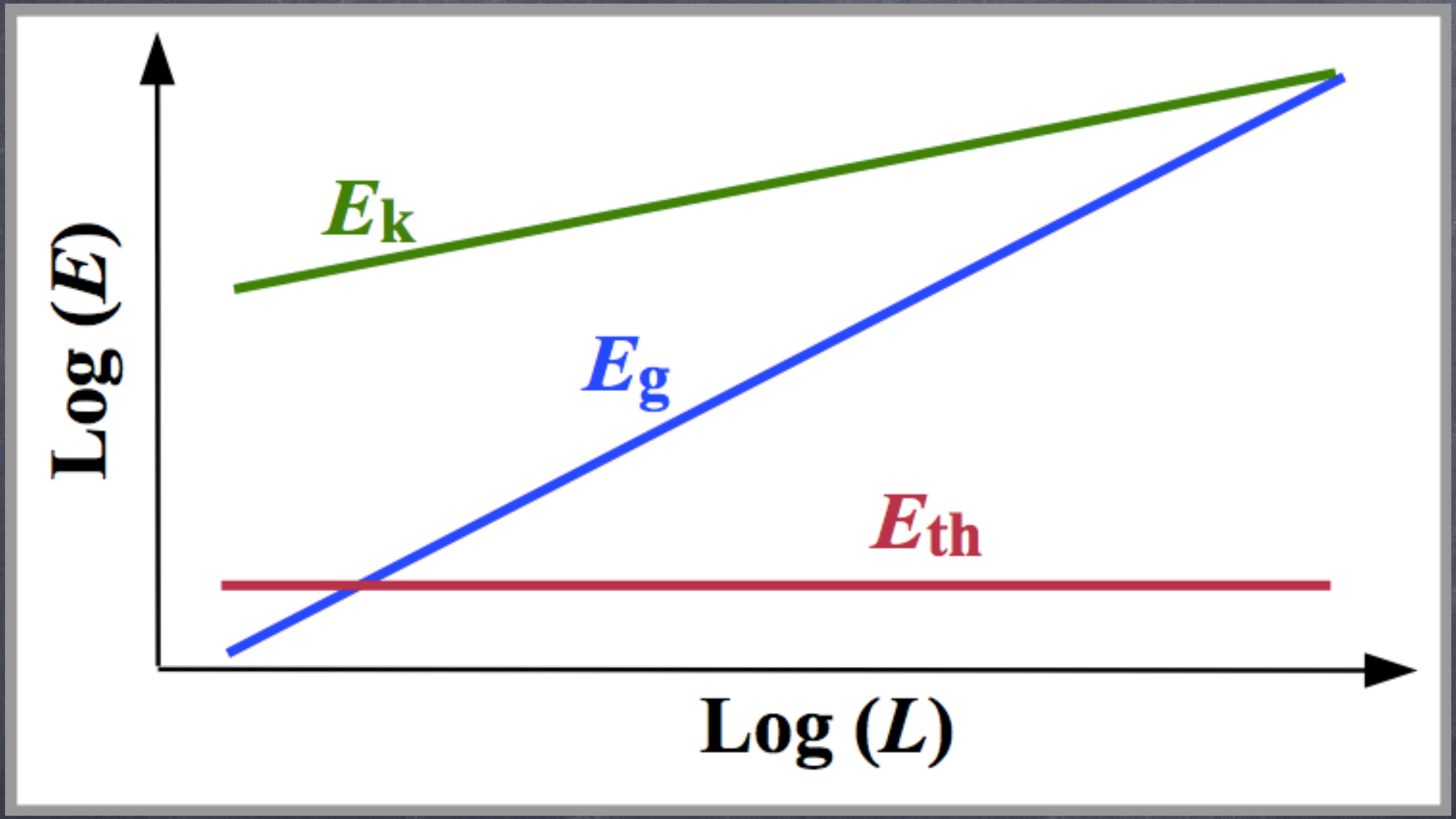
Reynolds number: $Re = UL / \nu \sim 10^8$

Sonic Mach number: $\mathcal{M}_s = \sigma_u / c_s \sim 20$

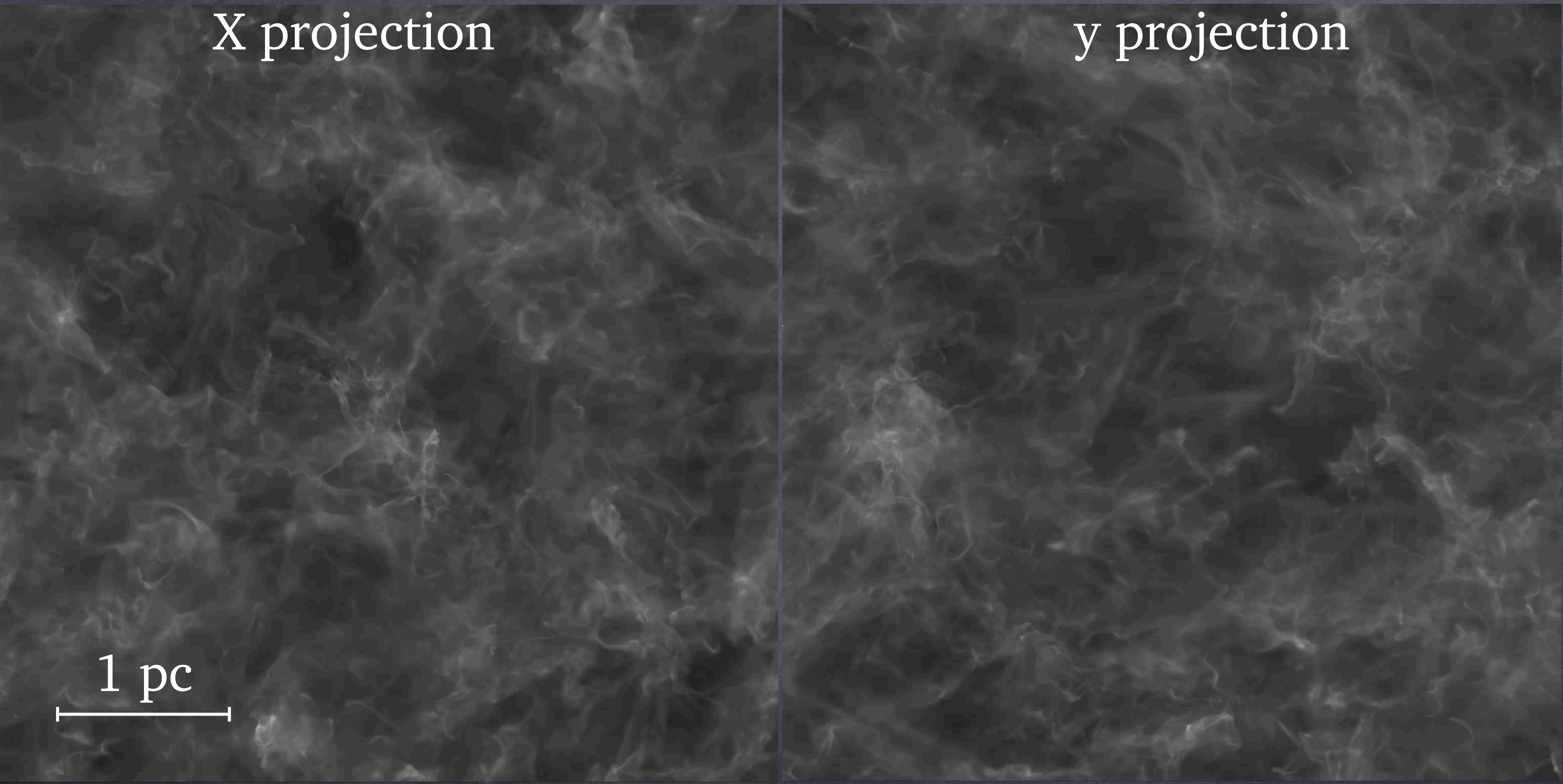


The turbulent energy is dominant on all but the largest scales.

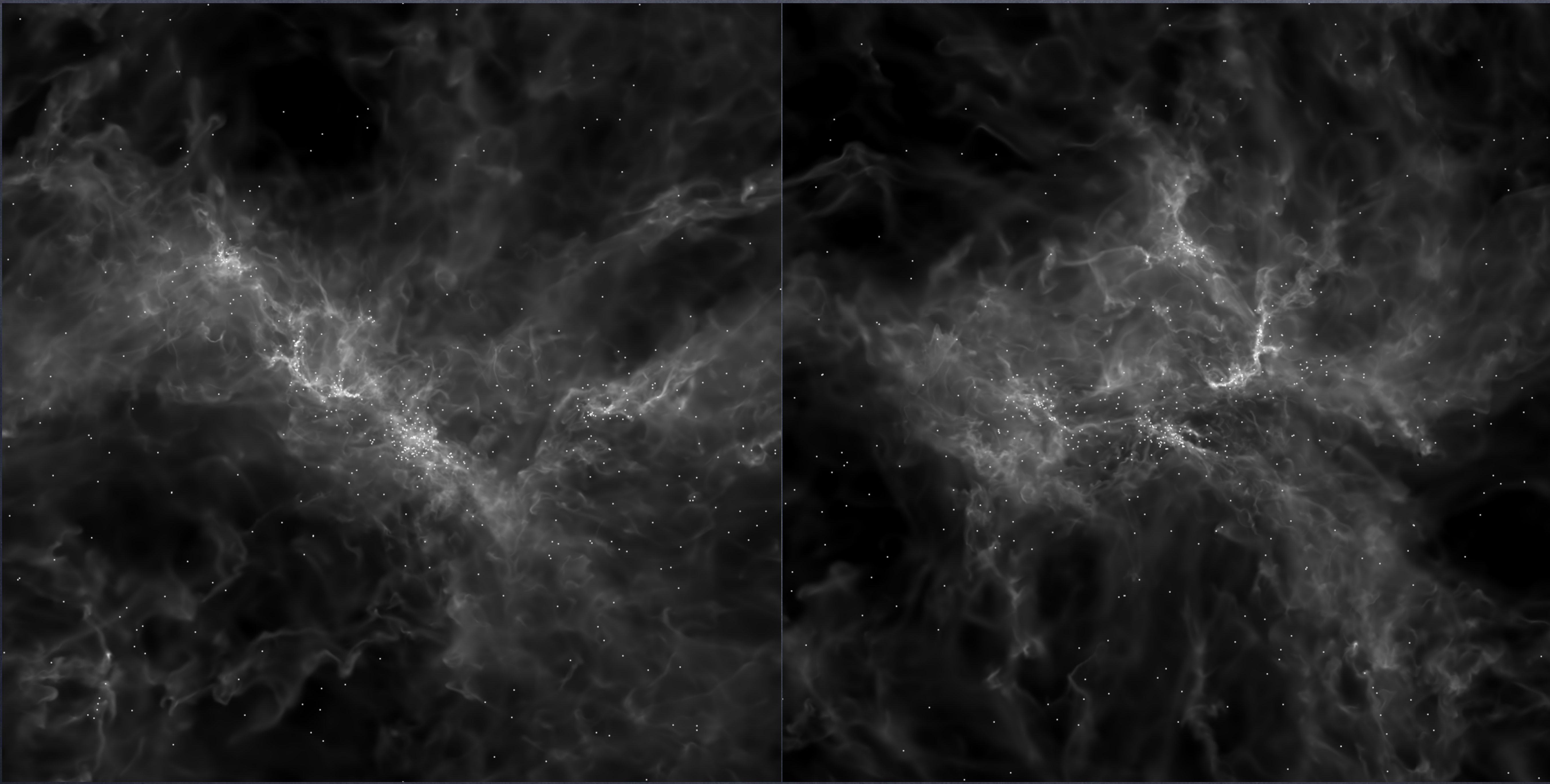
Energies per unit mass: $E_k \sim L$ $E_g \sim \rho\, L^2 \sim L^2$



- The turbulence can prevent the gravitational collapse
- But the turbulence also creates density enhancements
- Gravity dominates in density peaks —> star formation



Evolution during 3.2 Myr, from 1 to 1,300 stars



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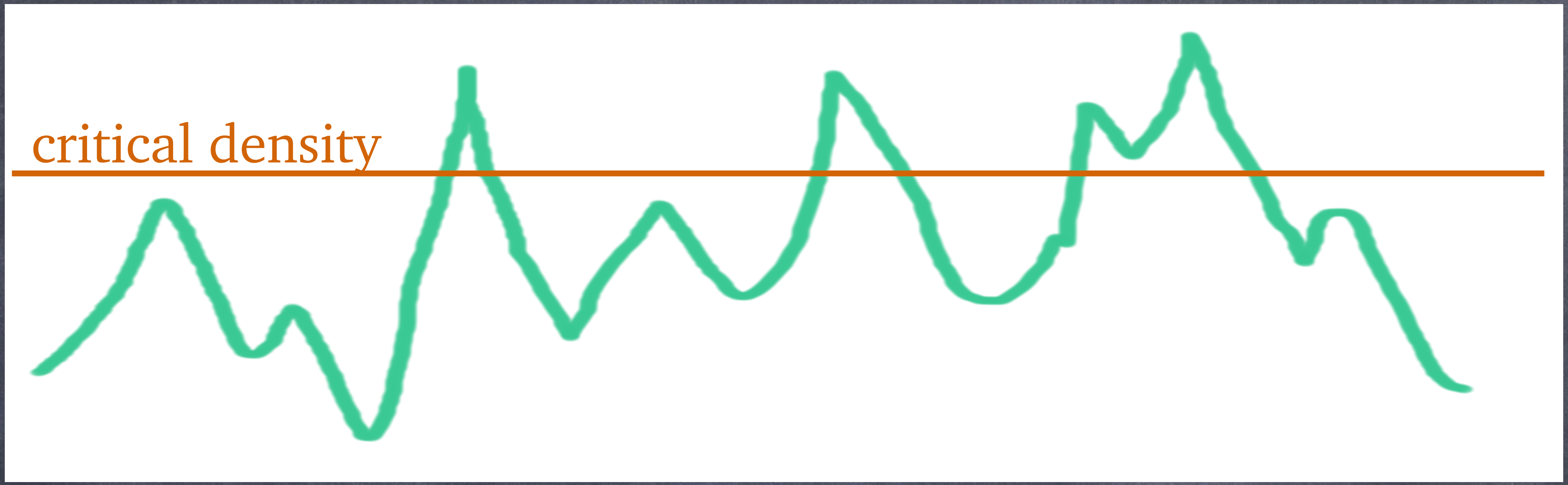
The SFR from Turbulent Fragmentation

- Take advantage of the universal statics of supersonic turbulence (density PDF and power spectrum, velocity scaling).
- Define a critical density for star formation, based on the ratio of the sonic scale and the Jeans length.
- Expressed the SFR as the integral of the PDF above the critical density, divided by the local free-fall time.

(Krumholz and McKee 2005; Padoan and Nordlund 2011; Chabrier and Hennebelle 2011; Federrath and Klessen 2012; Hopkins 2013)

Assuming the turbulence statistics are stationary, let's consider a snapshot in time:



Let's define a critical density above which the density fluctuations exceed the local Jeans mass.

The peaks above the critical density collapse in a free-fall time, so the SFR is given by the mass fraction above the critical density, divided by the free-fall time.

The Critical Density

The statistics of the density field (PDF and power spectrum) of supersonic turbulence have been determined with numerical simulations. They are universal and depend mainly on the rms Mach number (also on the magnetic field strength and the compressibility).

So we can scan the density field and identify all the density peaks that are gravitationally unstable, as in the Press-Schechter (*Chabrier and Hennebelle 2011*) or the excursion-set (*Hopkins 2013*) formalisms.

More intuitively, we can define the critical density as that of the critical Bonnor-Ebert sphere confined by the external turbulent pressure (*Padoan and Nordlund 2011*, related to, but different from *Krumholz and McKee 2005*):

$$\frac{\rho_{\rm cr}}{\rho_0} \propto f(\beta)\,\alpha_{\rm vir}\,\mathcal{M}_{\rm S}^2$$

Assuming a Larson line width-size relation, the expression simplifies into a constant, dependent only on the mean temperature (*Padoan et al. 2014*):

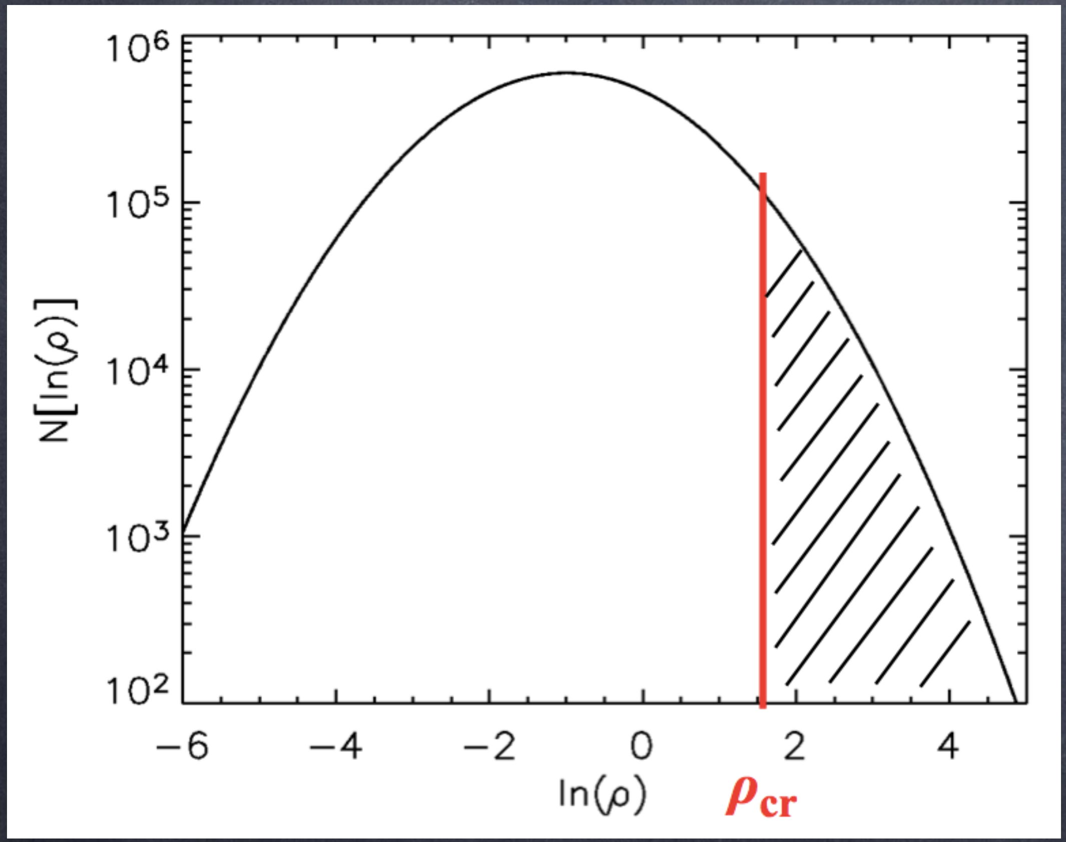
$$n_{\rm H,cr} \approx (5-10)\times 10^4 {\rm cm}^{-3}\,(T/10\,{\rm K})^{-1}$$

The Density PDF and the SFR

We know the density field of supersonic isothermal turbulence follows a universal PDF that is a Log-Normal and depends only on the rms Mach number of the turbulence, $\mathscr{M}_S$:

$$p(x)dx = \frac{x^{-1}}{(2\pi\sigma^2)^{1/2}} \exp\left[-\frac{(\ln x + \sigma^2/2)^2}{2\,\sigma^2}\right] dx \qquad \sigma^2 \approx \ln[1 + b^2 \mathcal{M}_S^2\, \beta/(\beta+1)]$$

(*Padoan and Nordlund 2011; Molina et al. 2011*)



The SFR is given by the integral of the PDF:

$$\mathrm{SFR}_{\mathrm{ff}} = \epsilon \int_{x_{\mathrm{cr}}}^{\infty} \frac{\tau_{\mathrm{ff},0}}{\tau_{\mathrm{ff,cr}}(x)}\, x\, p(x)\, dx, \qquad x = \frac{\rho}{\rho_0}$$

(*Hennebelle & Chabrier 11*; *Federrath & Klessen 12*)

The SFR depends on the three non-dimensional parameters: $\mathscr{M}_S$, β, α_{vir}

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Empirical SFR from Numerical Models

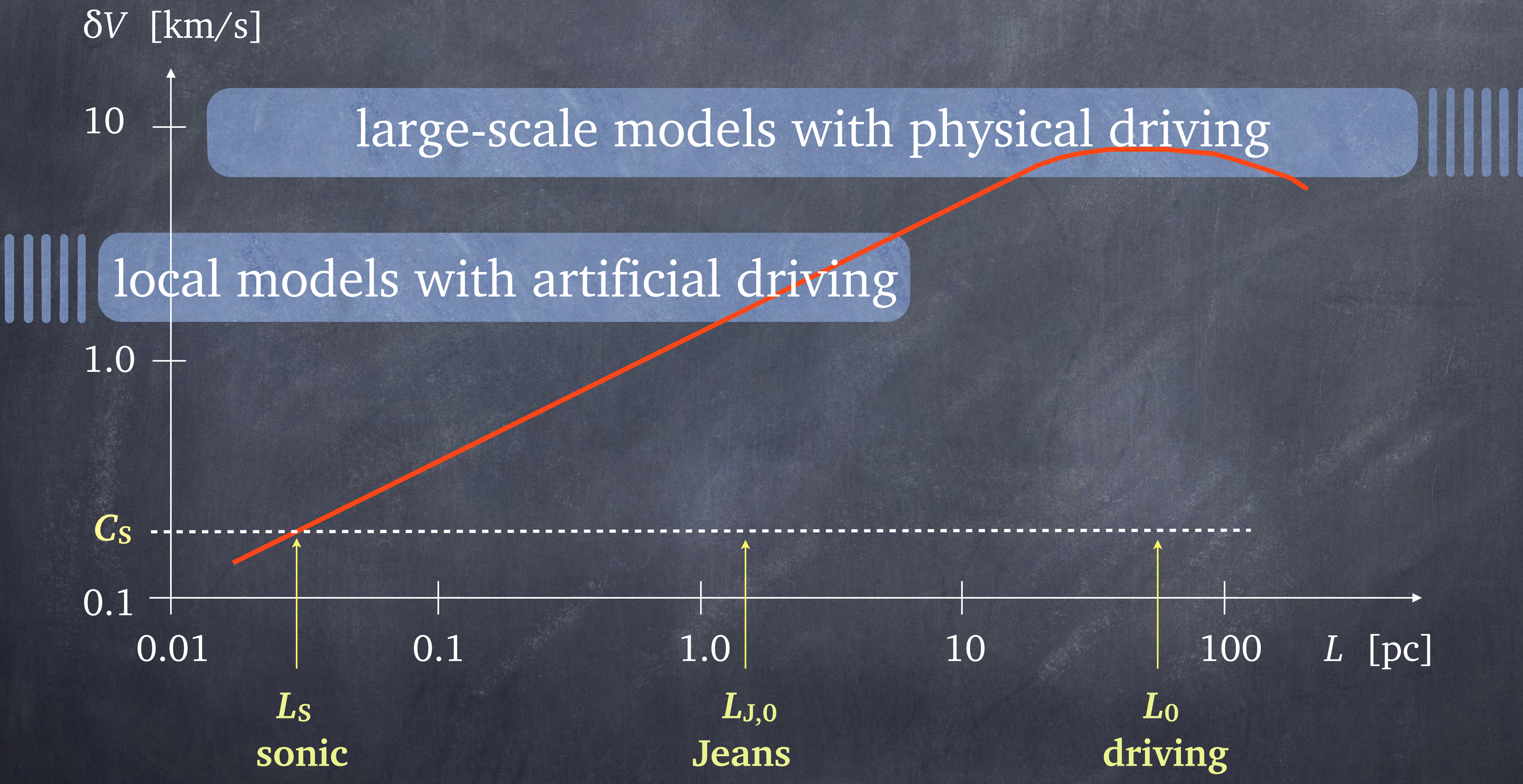
The fundamental length-scales, from large to small, are:

- driving scale, L_0 (largest turbulence turnover time)
- Jeans length, $L_{J,0}$ (gravity versus thermal pressure),
- sonic scale, L_S (turbulence versus thermal pressure),
- dissipation scale (smallest turbulence turnover time)

We must include $L_{J,0}$ and resolve L_S, because the SFR is controlled by $L_{J,0}/L_S$ (the SFR is low precisely because $L_S << L_{J,0}$)

Depending on the approach, we may or may not include the driving scale, L_0

The length-scales of star formation



Small-scale simulations with random driving (unigrid or AMR):

$[L_S\ , L_{J,0}] \rightarrow L_{\text{box}}/\mathrm{d}x \sim 10^3$

The limited range of scales is good for parameter studies
$\longmapsto$ derivation of the SFR law

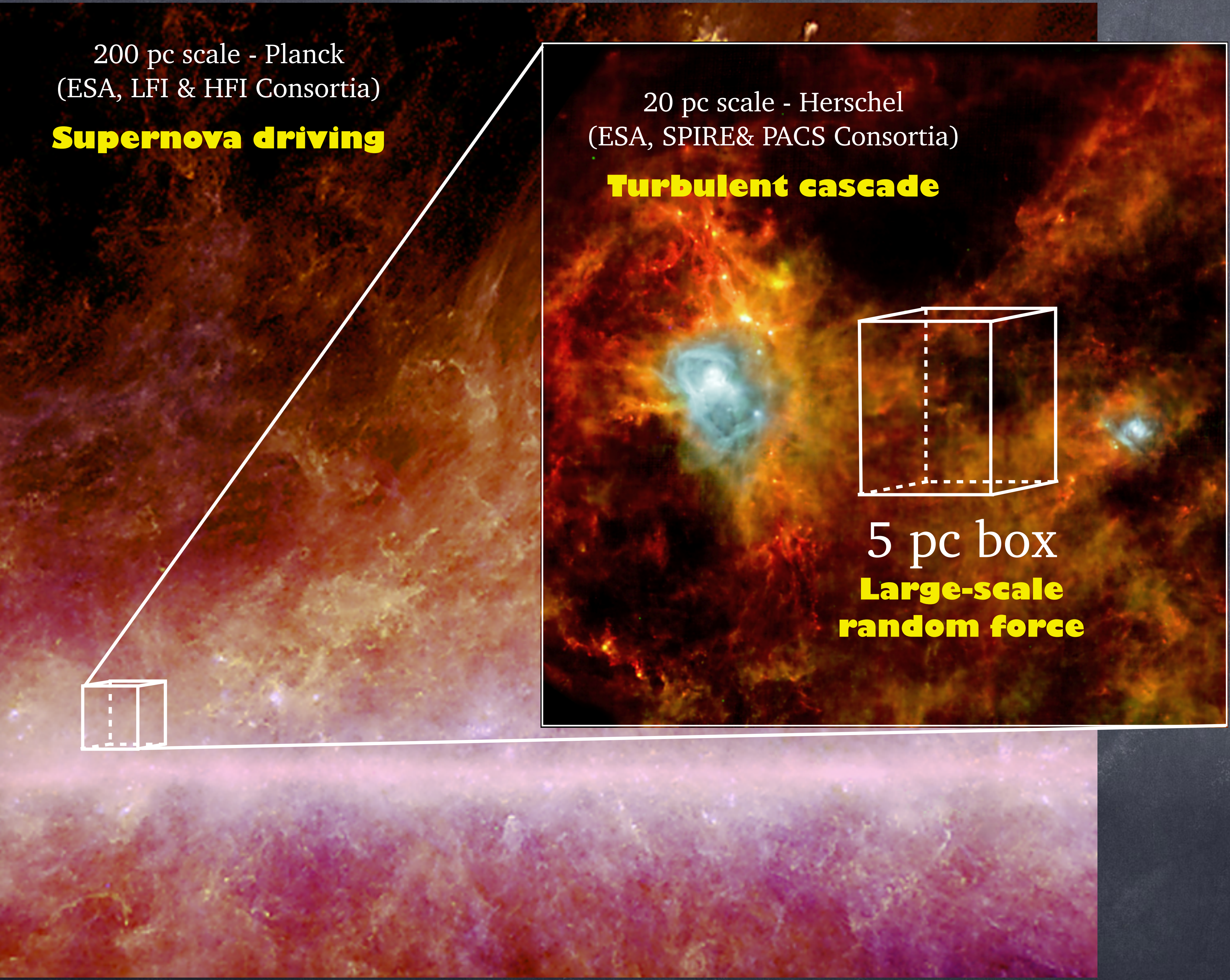
Large-scale simulations with physical (SN) driving (AMR):

$[L_S\ , L_0] \rightarrow L_{\text{box}}/\mathrm{d}x > 10^5$

The very large scale yields a large sample of star-forming regions, all with realistic boundary and initial conditions
$\longmapsto$ derivation of the intrinsic variance of the SFR law

$\longmapsto$ derivation of global SFR (self regulation?)

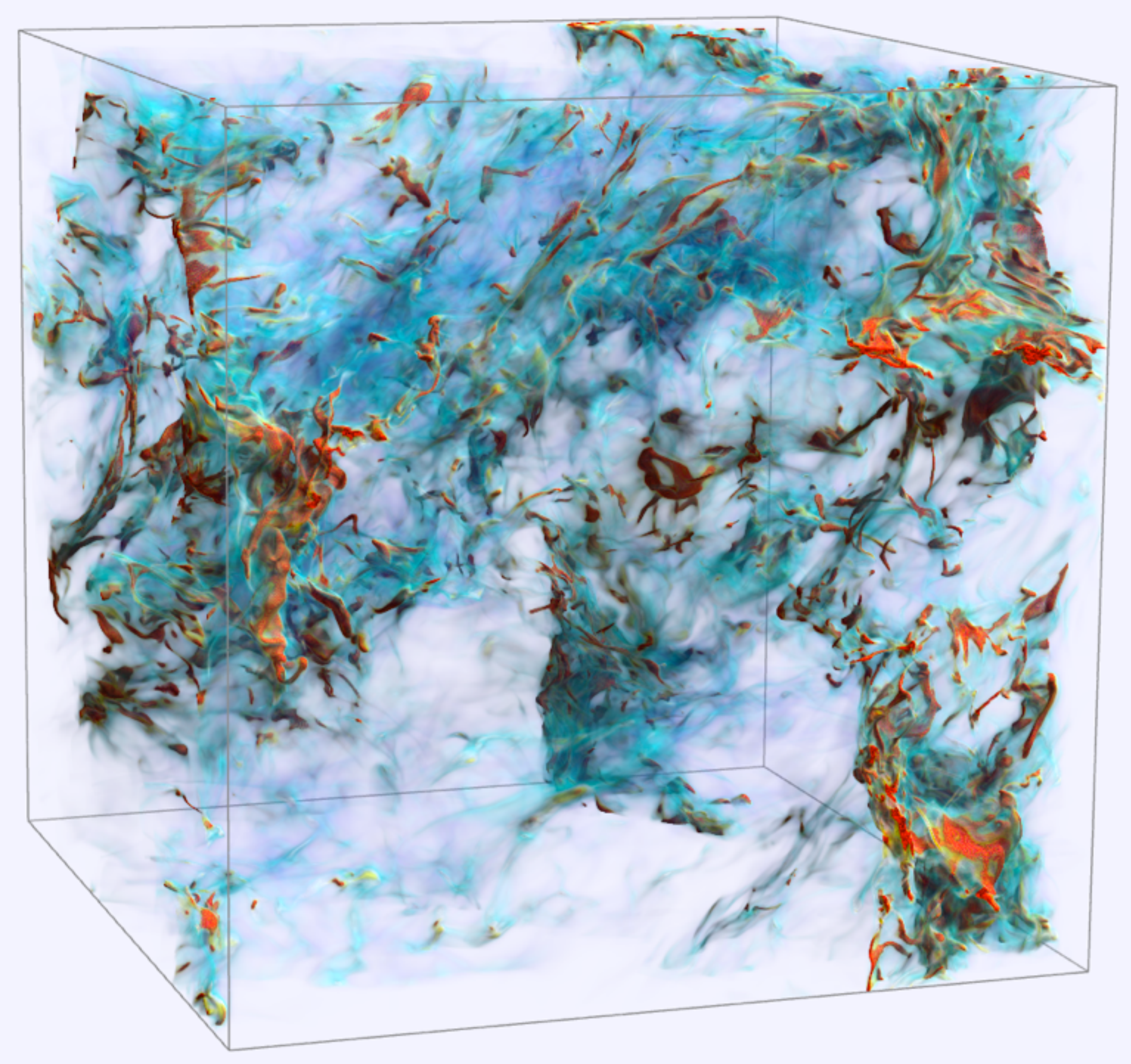
Local models below the physical driving scale



A chunk of a MC:

Periodic Box
Random forcing
Isothermal E.O.S.
Self-gravity
Sink particles

1000 $M_\odot$ in a 5 pc box - 60 AU resolution

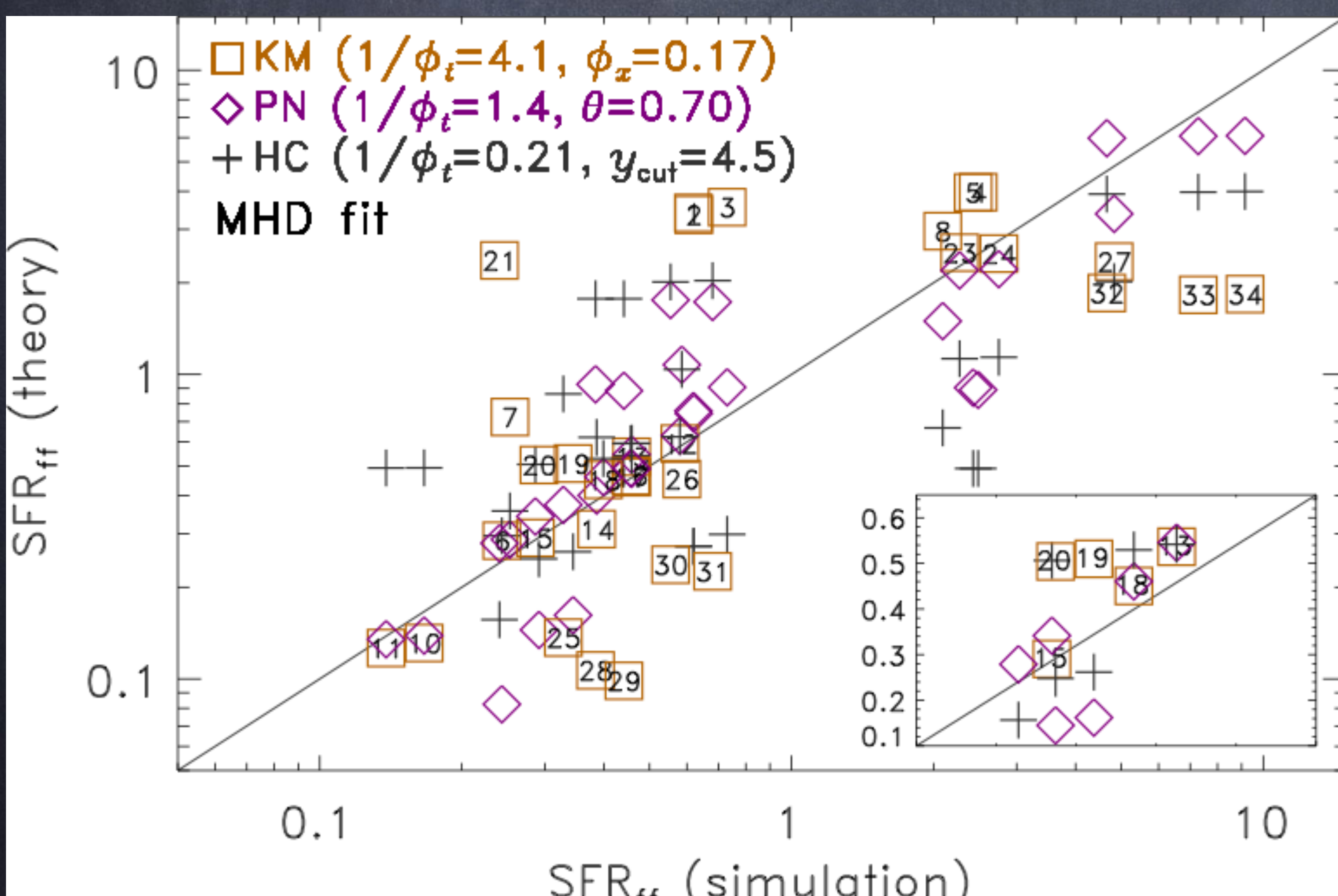


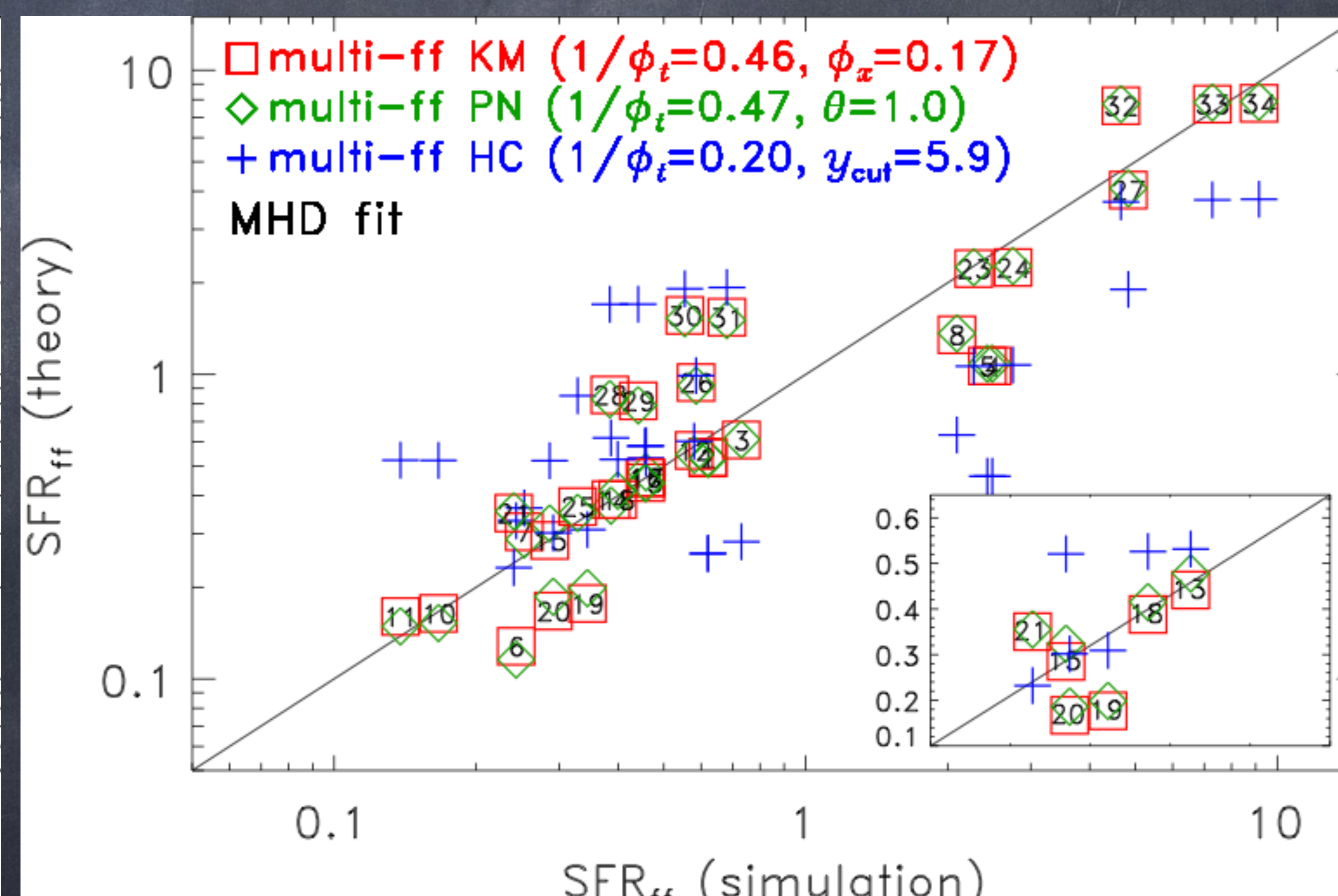
Small-Scale Simulations with Random Driving

Large parameter studies with such models have been recently carried out both with uniform-grid simulations (*Padoan and Nordlund 2011*) and with AMR (*Padoan, Haugbolle, Nordlund 2012; Federrath and Klessen 2012*).

The non-dimensional parameters controlling the SFR are $\alpha_{\rm vir}$, $\mathcal{M}_{\rm S}$ and $\mathcal{M}_{\rm A}$.

The numerical experiments roughly confirm the analytical models of the SFR, for a large range of values of $\alpha_{\rm vir}$, $\mathcal{M}_{\rm S}$ and $\mathcal{M}_{\rm A}$ (*Federrath and Klessen 2012*).

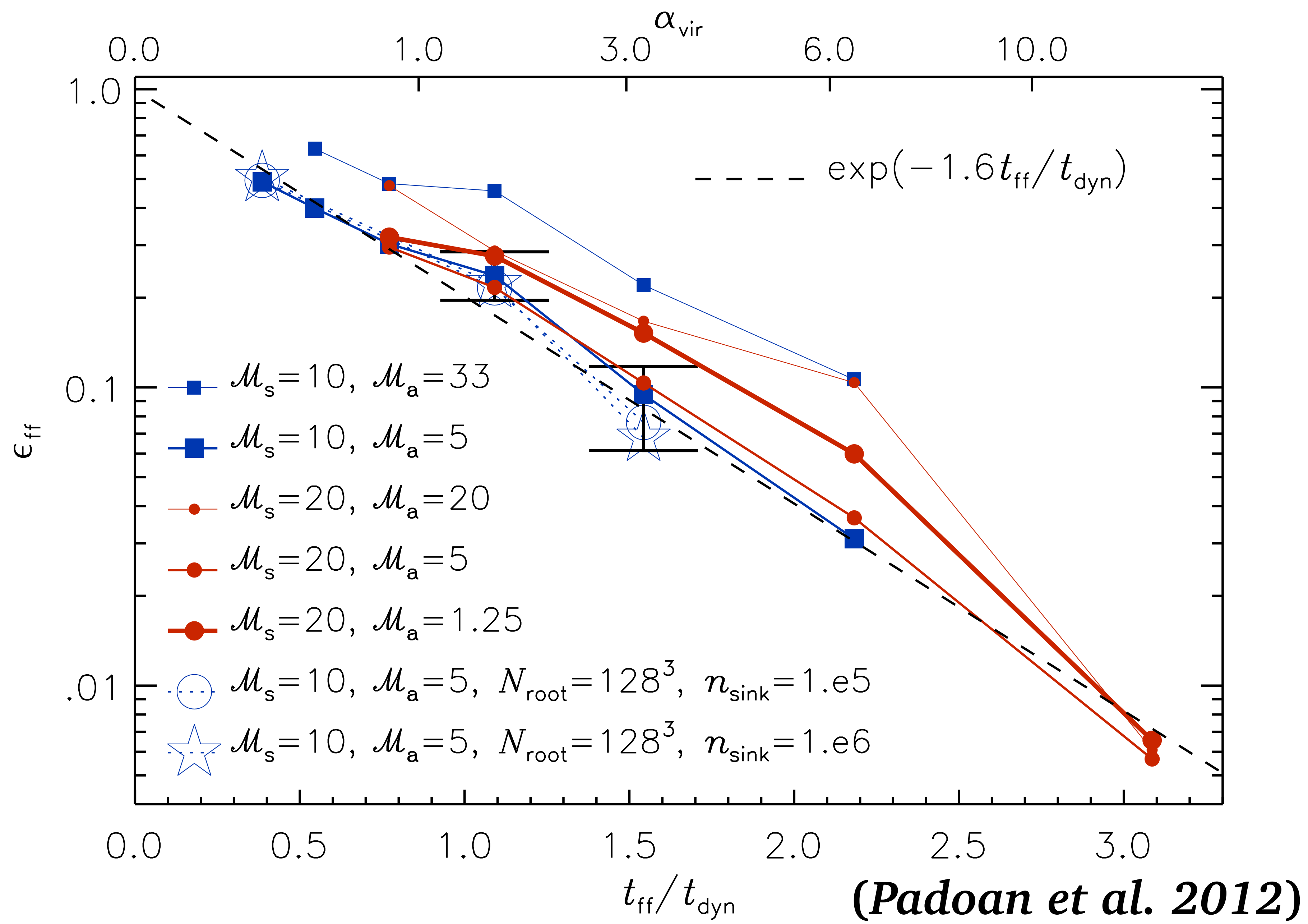




The most important of the the three parameters is $\alpha_{\rm vir}$ (*Padoan et al. 2012*)

$$\epsilon_{\rm ff} \sim \exp(-1.4\,\alpha_{\rm vir}^{1/2}) \sim \exp(-1.6\,t_{\rm ff}/t_{\rm dyn})$$

$$t_{\rm ff} = (3\pi/(32G\rho))^{1/2}, \qquad t_{\rm dyn} = R/\sigma_{\rm v,3D}$$



(Padoan et al. 2012)

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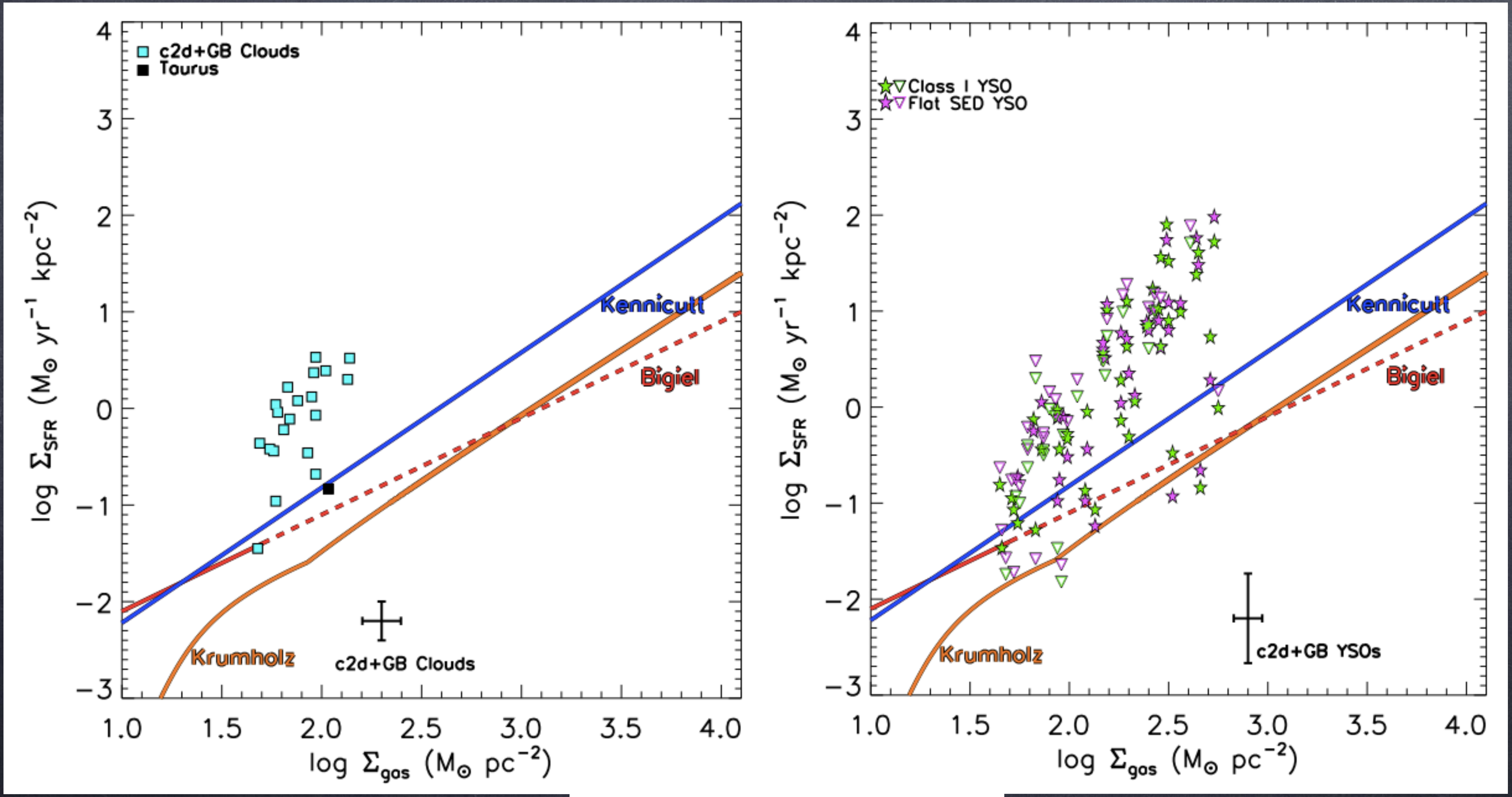
Global (Kennicutt), sub-kpc (Bigiel), and MC (Heiderman) scales are non-trivial to reconcile with each other:

- Different SFR probes: Hα, 24μm, stellar counts
- Extragalactic studies are blind to low mass stars



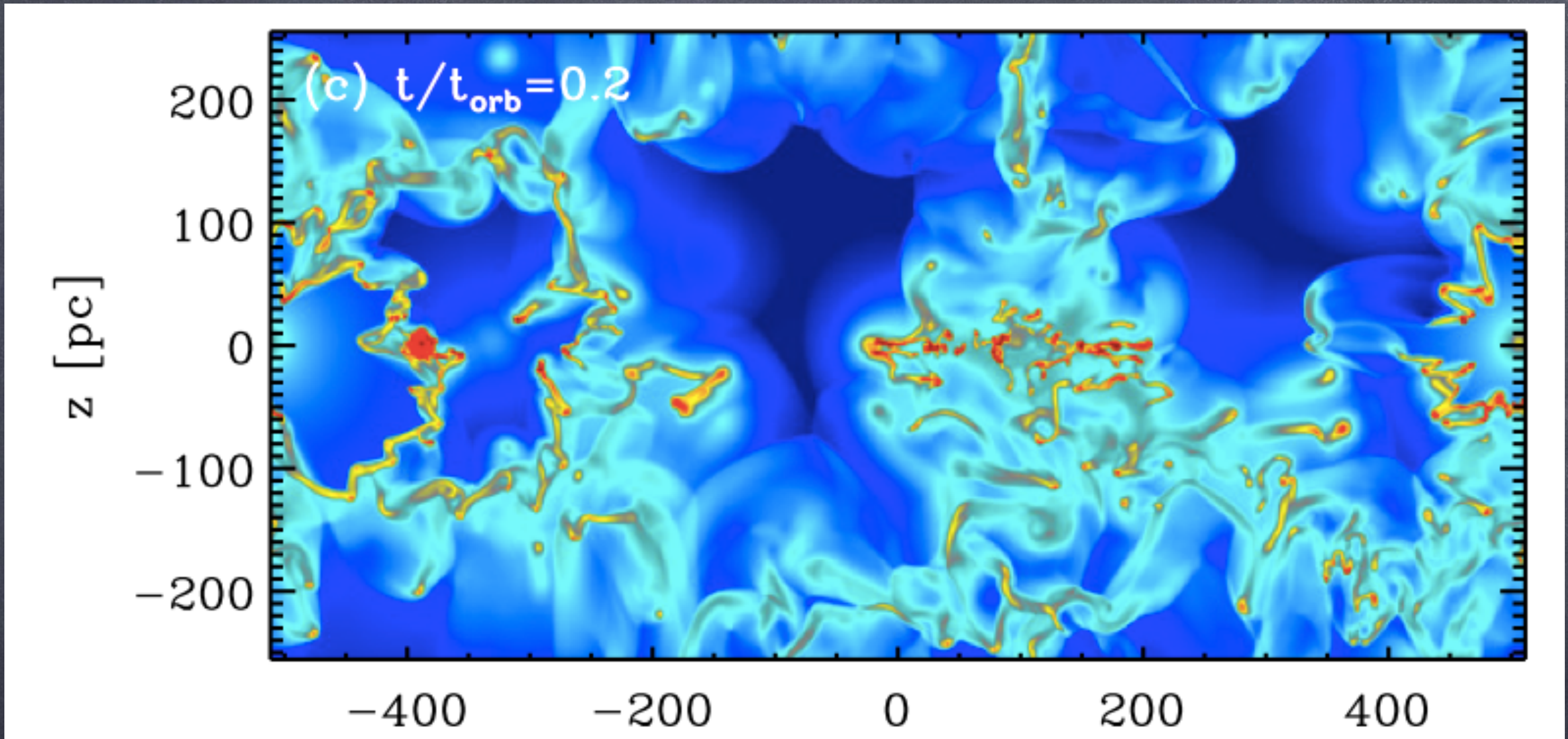
Heiderman et al. 2010

The analytical and the numerical SFR laws based on turbulent fragmentation can be applied to derive the Schmidt-Kennicutt relation of disk galaxies (e.g. *Krumholz and McKee 2005*; *Krumholz et al. 2012*; *Renaud et al. 2012*; *Federrath 2013*).

In the case of disk galaxies, the driving is mainly from SN explosions, and the driving scale is of order 100 pc. We need to know the spatial distribution of the main non-dimensional parameters, $\alpha_{\rm vir}$, $\mathscr{M}_{\rm S}$ and $\mathscr{M}_{\rm A}$, averaged over a 100 pc scale.

Self Regulation

Imposing disk vertical equilibrium and self-regulated star formation, one can derive the Schmidt-Kennicutt relation without even knowing the SFR law (e.g. *Ostriker et al. 2010*; *Ostriker and Shetty 2011*; *Kim et al. 2011*; *Ostriker et al. 2013*).



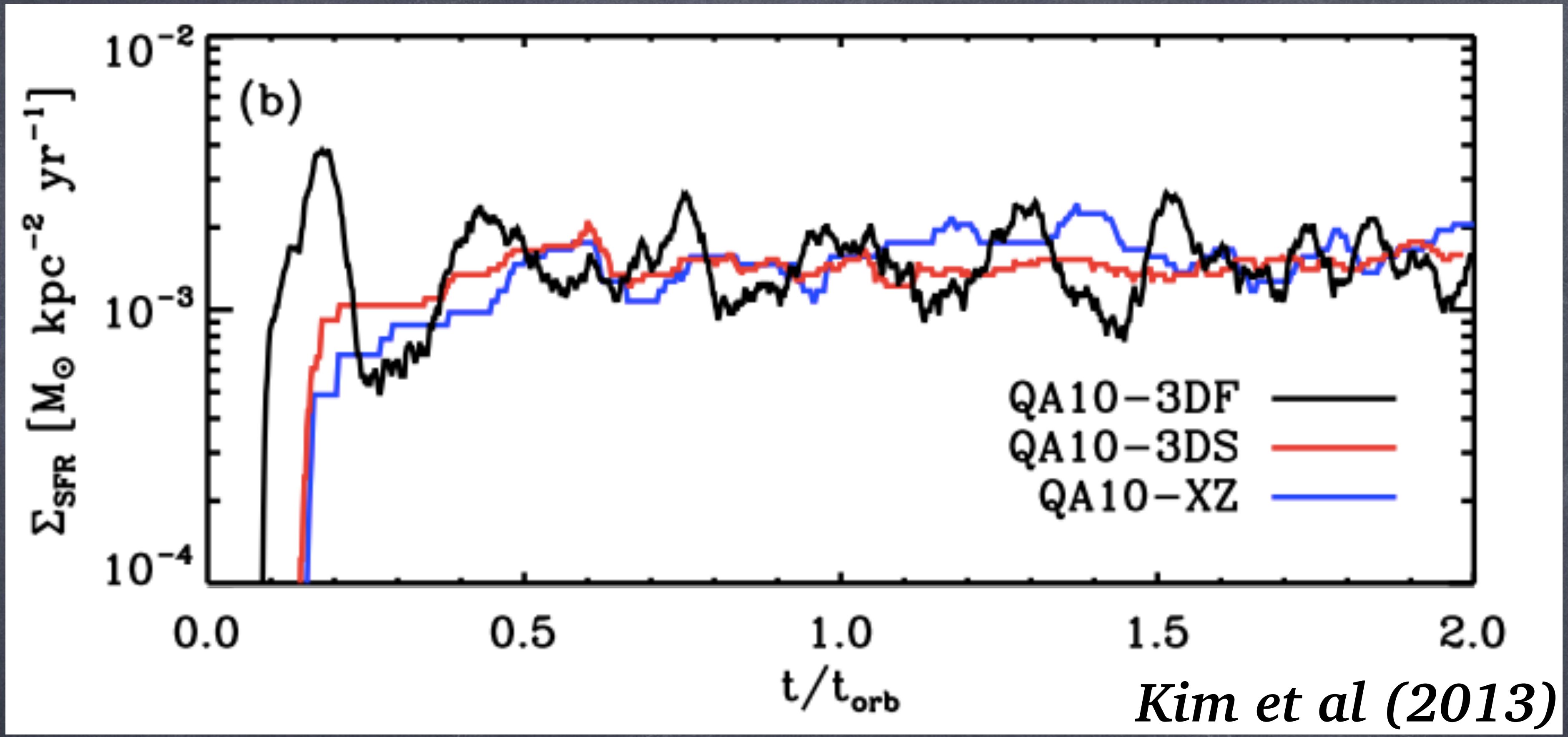
Kim et al. 2011

But is the SFR law derived from turbulent fragmentation consistent with self-regulation?

Self-Regulation

We have found that the SFR is very sensitive to α_{vir}. Since the SN driving that determines α_{vir} is proportional to the SFR, the process can self-regulate:

Larger SFR —> increased α_{vir} —> decreased SFR

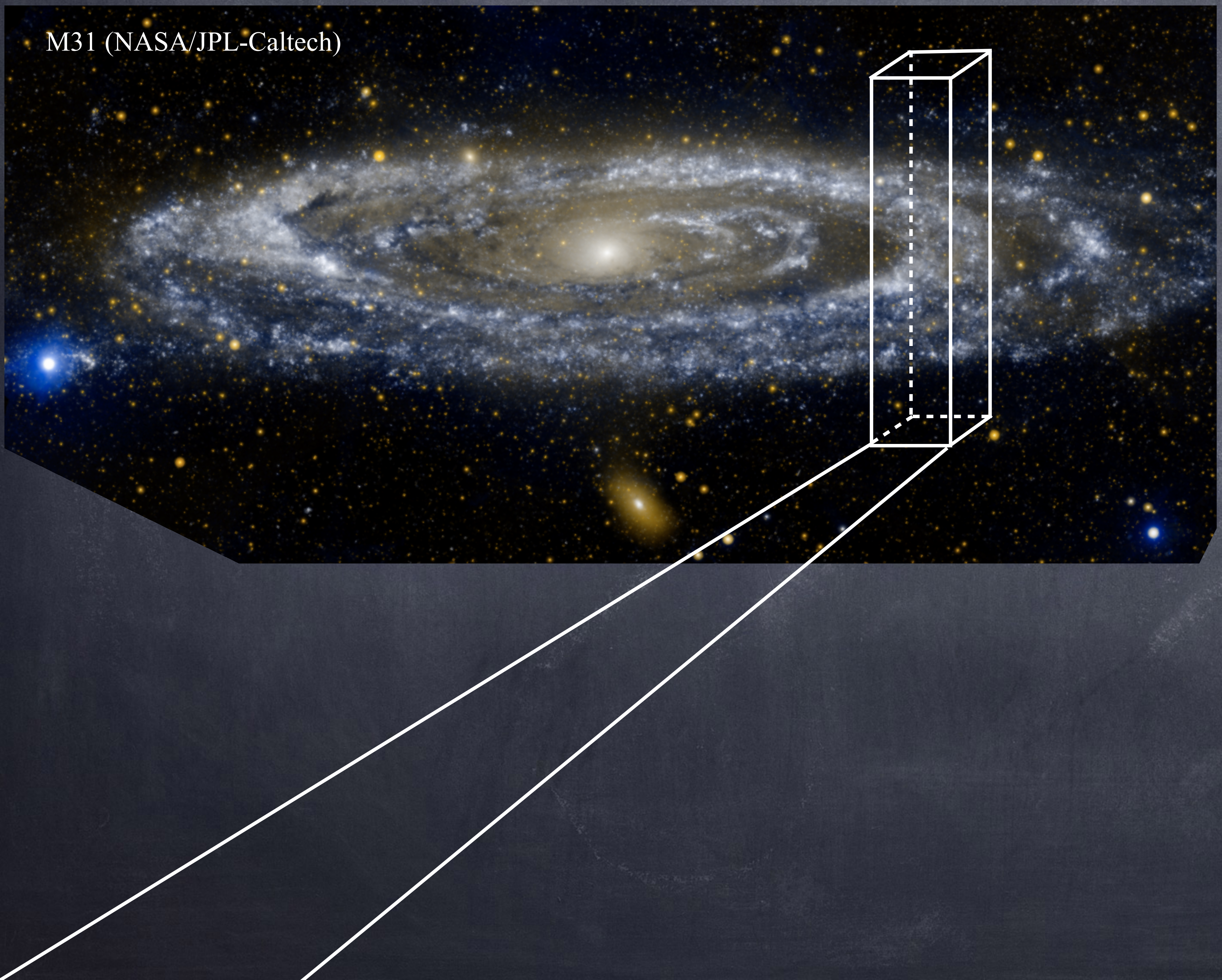


Kim et al (2013)

Using our simple exponential law, we derive an equilibrium value of $\alpha_{\mathrm{vir}} \sim 1$, giving a realistic gas consumption time of ~ 1 Gyr.

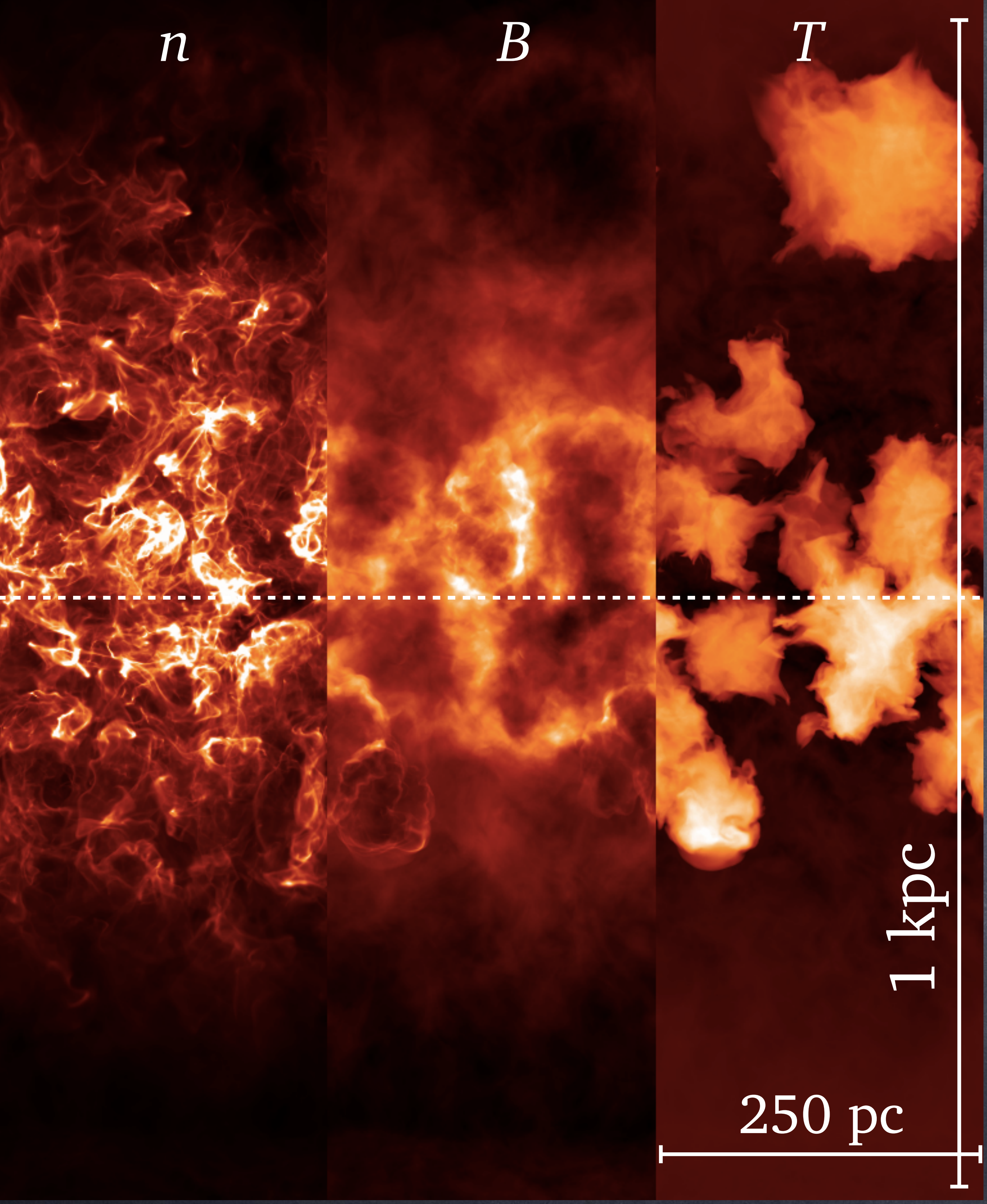
We can address the self-regulation numerically, with simulations including SN driving, while also resolving the formation of individual stars.

Large-Scale Models with Realistic Driving



A chunk of a galaxy:

Supernova driving
Heating and cooling
Galactic potential
Self-gravity
Sink particles



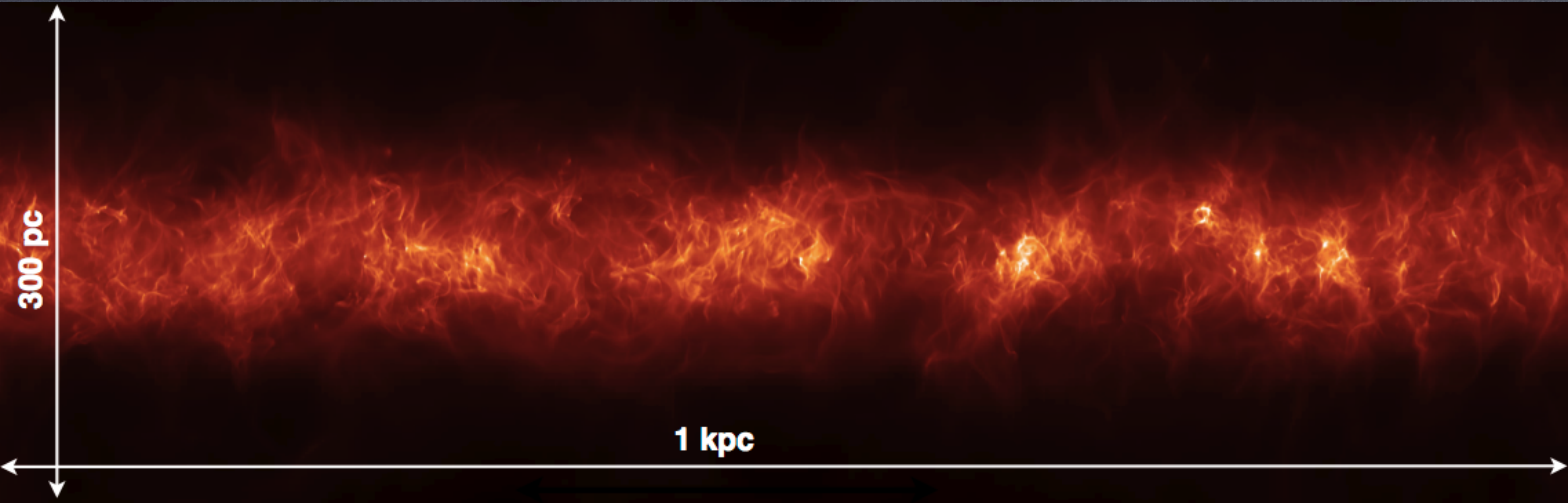
Range of scales:

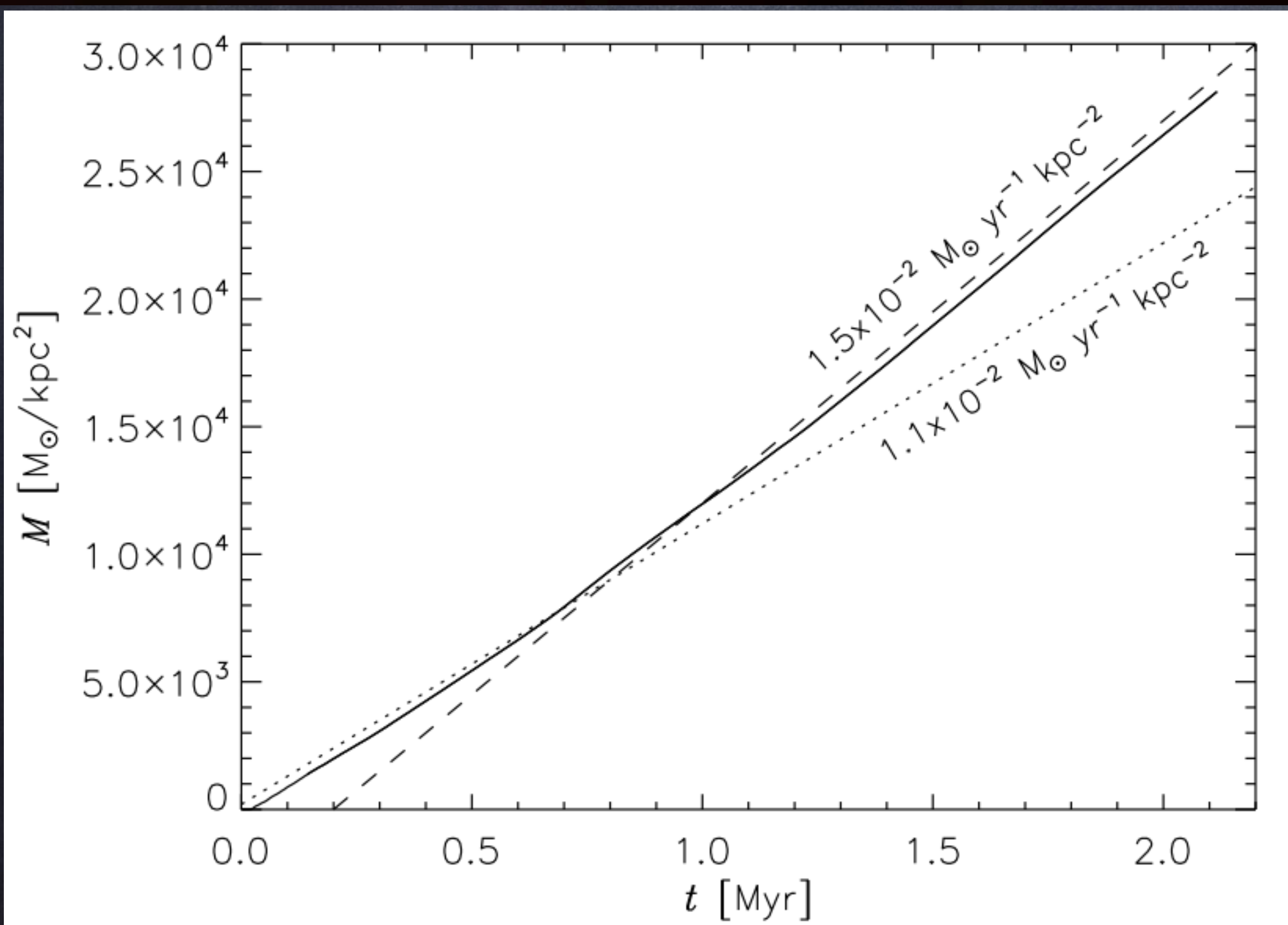
1-32 kpc - 10^{-2} pc

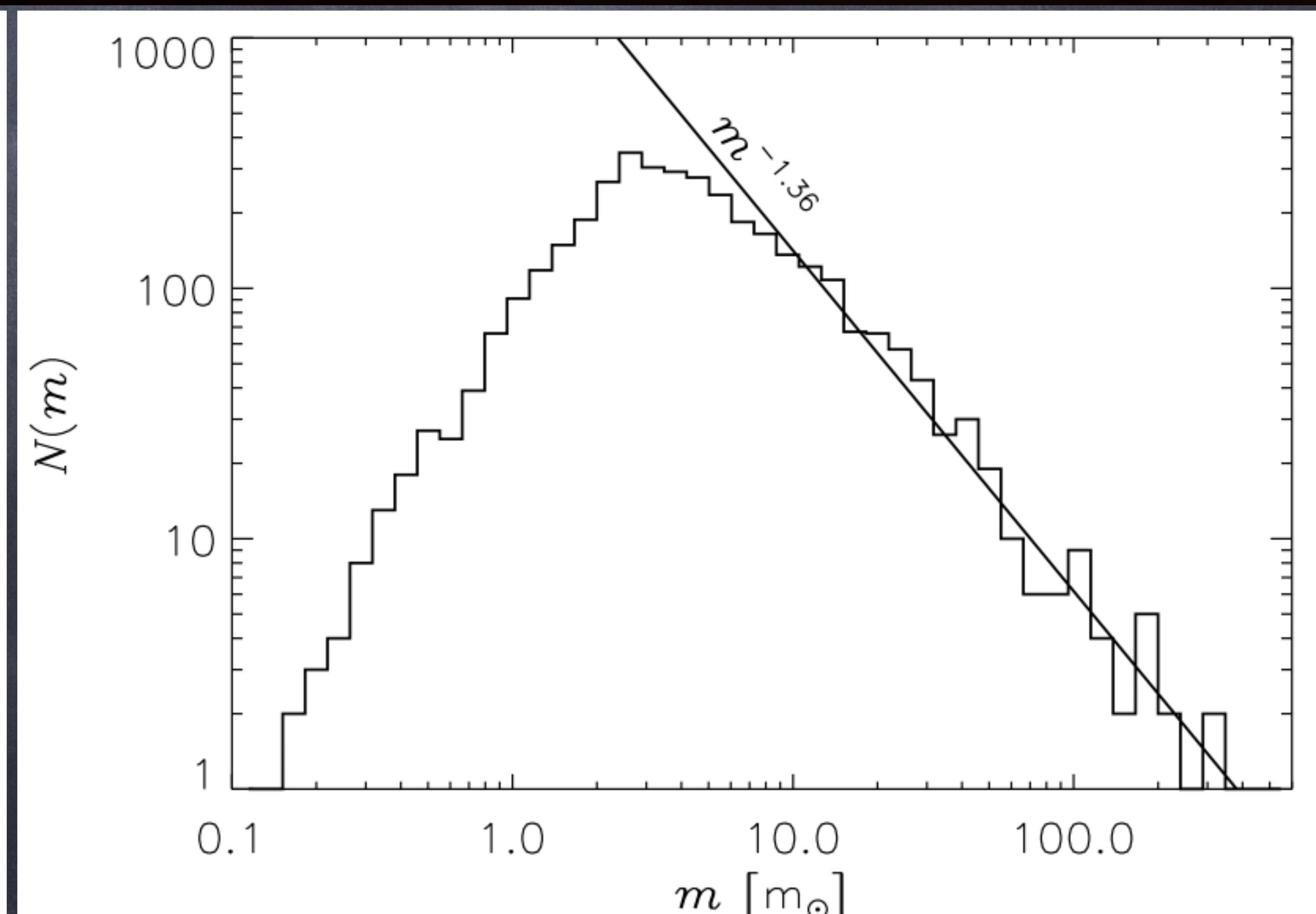
(17M cpu hr, PRACE on Supermuc)

- The SFR can be measured in many star-forming regions, with different α_{vir}, $\mathcal{M}_S$, $\mathcal{M}_A$
- Realistic initial and boundary conditions for each star-forming region
- We can derive the SFR law, SFR = f(α_{vir}, $\mathcal{M}_S$, $\mathcal{M}_A$)

Formation of 4,000 stars over 2 Myr, with realistic SFR and Salpeter IMF







Conclusions

- We understand how turbulent fragmentation leads to the observed low star formation rate, and the SFR law has been modeled based on the statistics of supersonic turbulence
- The SFR law only depends on the three main non-dimensional parameters of the turbulence: α_{vir}, $\mathcal{M}_{\mathrm{S}}$ and $\mathcal{M}_{\mathrm{A}}$ and is applicable to any scale (below the driving scale of the turbulence)
- The numerical parameter studies confirm the validity of the analytical model. They also suggest an empirical SFR law that depends mainly on α_{vir}
- The sensitive dependence of the SFR on α_{vir} explains the approximate self-regulation of star formation in disk galaxies.
- Global numerical models including the SN driving scale confirm the values of the self-regulated SFR and α_{vir}.