Optical modeling of standing waves in HIFI.

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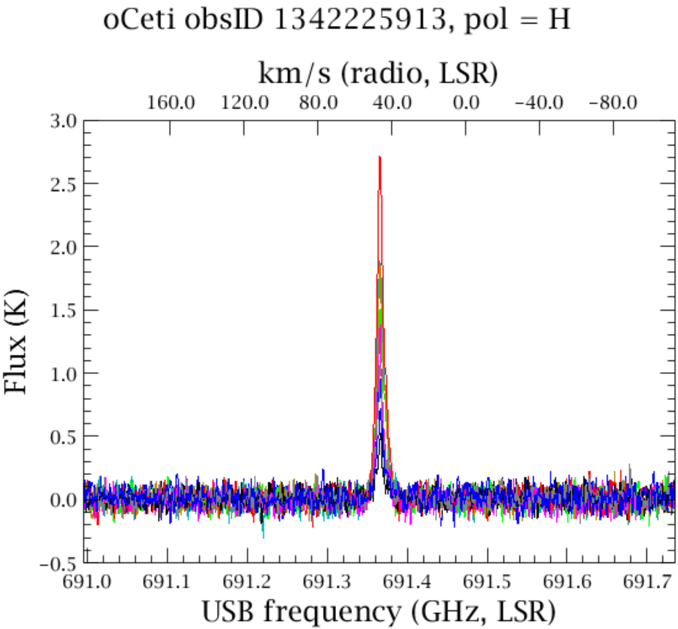
Outline

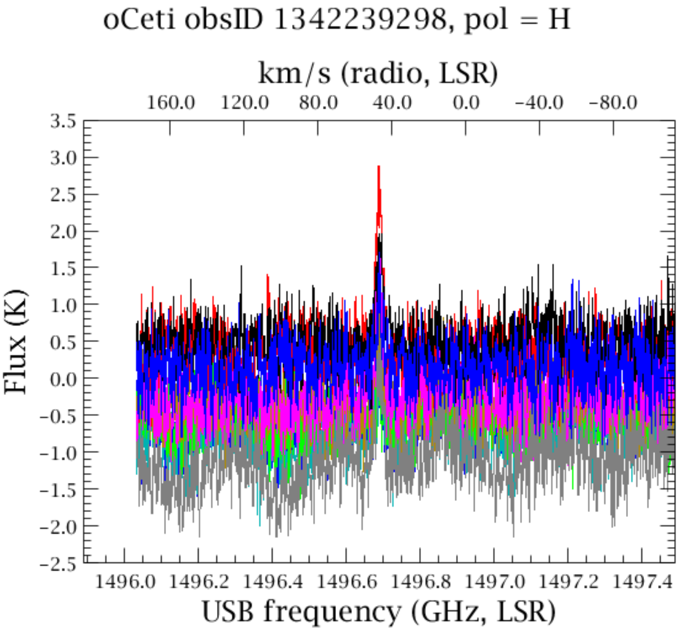
Part I

Origins and effects of standing waves

Outline

1. Coherence.
2. Wavelength dependency.
3. Calibration scheme fails.


oCeti obsID 1342225913, pol = H
km/s (radio, LSR)
160.0 120.0 80.0 40.0 0.0 -40.0 -80.0
3.0 2.5 2.0 1.5 1.0 0.5 0.0 −0.5
Flux (K)
691.0 691.1 691.2 691.3 691.4 691.5 691.6 691.7
USB frequency (GHz, LSR)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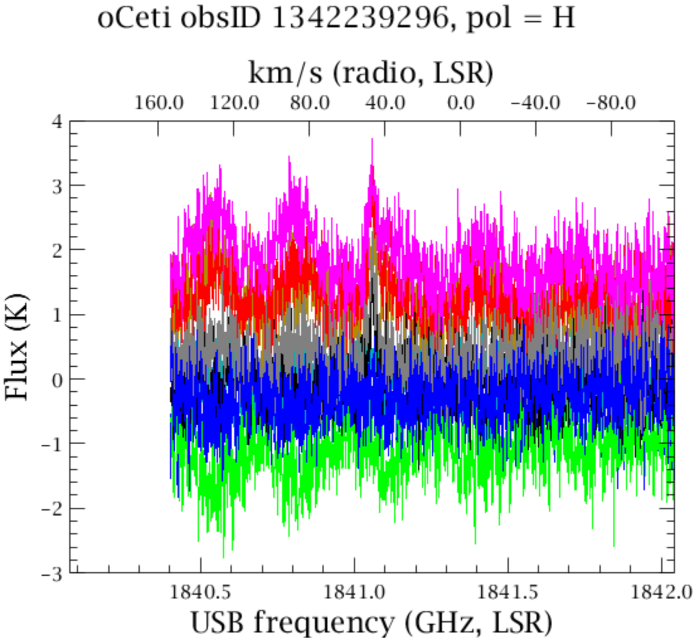
km/s (radio, LSR)
160.0
120.0
80.0
40.0
0.0
-40.0
-80.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
Flux (K)
1496.0
1496.2
1496.4
1496.6
1496.8
1497.0
1497.2
1497.4
USB frequency (GHz, LSR)


oCeti obsID 1342239296, pol = H
km/s (radio, LSR)
160.0
120.0
80.0
40.0
0.0
-40.0
-80.0
Flux (K)
4
3
2
1
0
-1
-2
-3
1840.5
1841.0
1841.5
1842.0
USB frequency (GHz, LSR)

Outline

1. Coherence.
2. Wavelength dependency.
3. Calibration scheme fails.

Recipe for standing waves.

Two ingredients.

$$\text{reflections} + \text{coherence} = \begin{cases} \text{standing waves} \\ \text{interferences} \\ \text{fringes} \\ \text{ripples} \end{cases}$$

The wave interferes with its reflections.

Is HIFI coherent?

- Local oscillator line: yes, by construction.
- Sky, LO noise, calibration black bodies: a priori no (thermal).

Coherence length for $\Delta f = 1\,\mathrm{MHz}$ at $f = 1\,\mathrm{THz}$

Radio definition.

$$L = \frac{c}{\Delta f}$$

$$L \approx 300\,\mathrm{m}$$

Optical definition.

$$L = \frac{2\ln(2)}{\pi n}\frac{\lambda^2}{\Delta\lambda}$$

$$L \approx 100\,\mathrm{m}$$

Coherence achieved.

Longest cavity: $1.5\,\mathrm{m}$. After 10 travels: $15\,\mathrm{m} \ll L$.

The noise is coherent enough to interfere.

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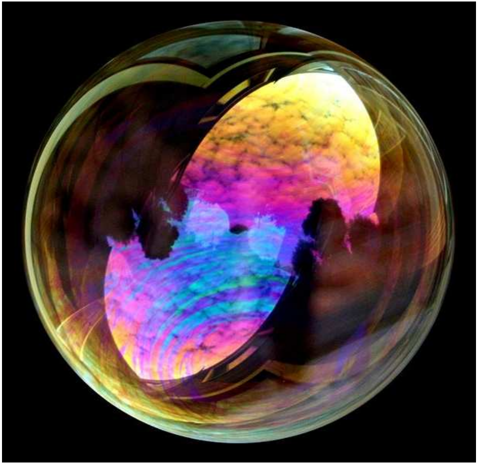
Coherence achieved.

Longest cavity: $1.5\,\mathrm{m}$. After 10 travels: $15\,\mathrm{m} \ll L$.

The noise is coherent enough to interfere.

Not unusual.

Natural light, a priori incoherent, interferes anyway in a soap bubble.



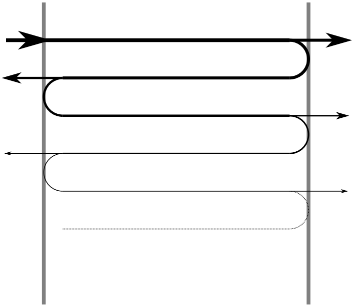
Outline

1 Coherence.

2 Wavelength dependency.

3 Calibration scheme fails.

Infinite reflections.



Their sum converges: $$\sum_{i=0}^{\infty} r^i = \frac{1}{1-r}$$

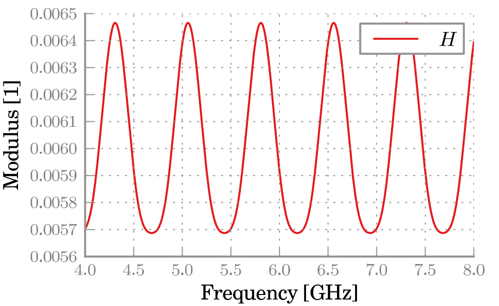
Wavelength-dependent.

Standing wave pattern.

Period and cavity length.

$$T_{\text{standing wave}} = \frac{c}{2L_{\text{cavity}}}$$

NOT A SINE.



High-quality cavities have narrow resonnance frequencies.

Outline

1 Coherence.

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3 Calibration scheme fails.

Standing waves should calibrate out.

3-points calibration (off, hot, cold):

$$P = K\frac{(P_{\text{on}}G_{\text{on}} + P_{\text{LO}}G_{\text{LO}}) - (P_{\text{off}}G_{\text{off}} + P_{\text{LO}}G_{\text{LO}})}{(P_{\text{hot}}G_{\text{hot}} + P_{\text{LO}}G_{\text{LO}}) - (P_{\text{cold}}G_{\text{cold}} + P_{\text{LO}}G_{\text{LO}})}.$$

If P_{LO} is constant
and $G_{\text{on}} = G_{\text{off}} = G_{\text{hot}} = G_{\text{cold}}$
then all the Gains disappear:

$$P = K\frac{P_{\text{on}} - P_{\text{off}}}{P_{\text{hot}} - P_{\text{cold}}}.$$

The continuum should not show a standing wave pattern.

Why the calibration does not clean the continuum.

Reason 1.

The calibration black bodies are not black enough.

They introduce cavities that do not exist on the sky path.

$G_{\text{hot}} \neq G_{\text{cold}} \neq G_{\text{sky}}$.

$$P = K \frac{(P_{\text{on}} - P_{\text{off}})\,G_{\text{sky}}}{(P_{\text{hot}}\,G_{\text{hot}}) - (P_{\text{cold}}\,G_{\text{cold}})}.$$

Why the calibration does not clean the continuum.

Reason 2.

The LO power is not always stable.

P_{LO} differs for each spectrum entering the calibration.

$$P = K\frac{(P_{\text{on}}\,G_{\text{on}} + P_{\text{LO1}}\,G_{\text{LO}}) - (P_{\text{off}}\,G_{\text{off}} + P_{\text{LO2}}\,G_{\text{LO}})}{(P_{\text{hot}}\,G_{\text{hot}} + P_{\text{LO3}}\,G_{\text{LO}}) - (P_{\text{cold}}\,G_{\text{cold}} + P_{\text{LO4}}\,G_{\text{LO}})}.$$

Why the calibration does not clean the continuum.

Reason 3.

The reflectivity of the mixer changes.

Mixer biased with a fixed voltage U.

Mixer current I varies with LO power.

Impedance $Z = U/I$ varies with LO power.

Reflection coefficient $\Gamma = \frac{Z-Z_s}{Z+Z_s}$ varies with LO power.

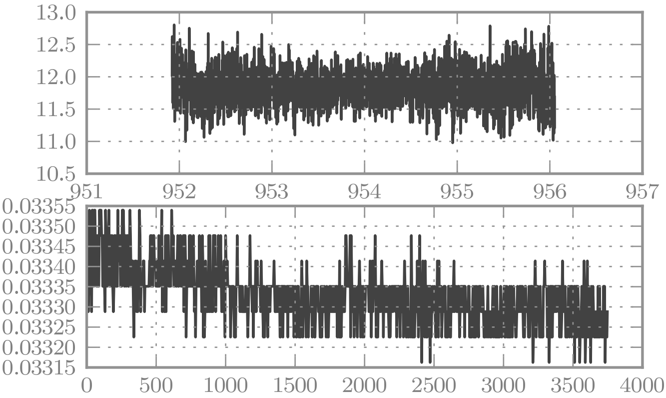
All the gains differ.

$$P = K\frac{(P_{\text{on}}\,G_{\text{on}} + P_{\text{LO1}}\,G_{\text{LO1}}) - (P_{\text{off}}\,G_{\text{off}} + P_{\text{LO2}}\,G_{\text{LO2}})}{(P_{\text{hot}}\,G_{\text{hot}} + P_{\text{LO3}}\,G_{\text{LO3}}) - (P_{\text{cold}}\,G_{\text{cold}} + P_{\text{LO4}}\,G_{\text{LO4}})}.$$

Continuum of Mars, band 3.

Taken from a map of Mars, the LO had thermalized.

Continuum.

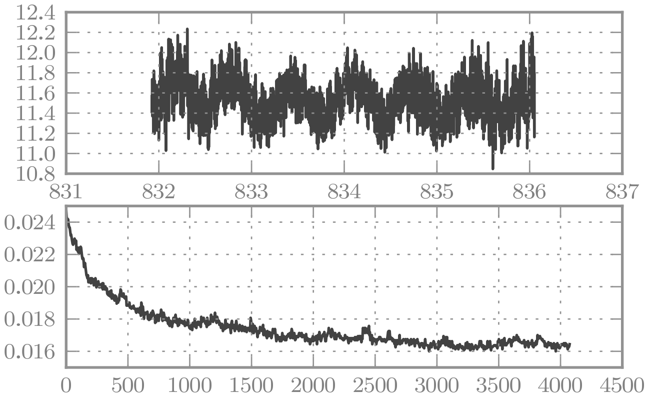


Mixer current vs time during the entire mapping observation.

Continuum of Mars, band 3.

Taken from another map of Mars, LO not thermalized yet.

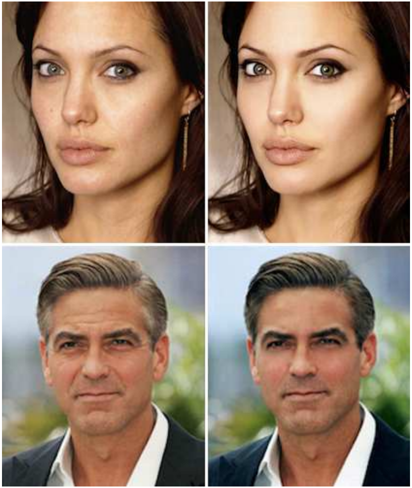
Continuum.



Mixer current vs time during the entire mapping observation.

How do we clean that?

Fit-subtract sines is not science. It is photoshop.



Part II

The model.

Outline.

4 Wave properties.

5 Representing a network.

6 Cascading networks.

7 Ellipsis.

8 Conclusion.

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Wave properties that matter.

Mandatory.

- Direction of propagation.
- Amplitude or power.
- Polarization.
- Phase.

Nice to have.

- Shape of the wave front.

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Scattering matrices handle ports.

Model the transfer of intensity and phase between network ports.

Scattering matrix.

Scattering matrix of an n-ports network.

$$\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} S_{1,1} & S_{1,2} & \dots & S_{1,n} \\ S_{2,1} & S_{2,2} & \dots & S_{2,n} \\ \dots & \dots & \ddots & \dots \\ S_{n,1} & S_{n,2} & \dots & S_{n,n} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$

Drawback.

All the parameters are scalar.

$\implies$ Does not understand polarization unless triple the ports.

Making scattering matrices aware of polarization.

I replace the scalar parameters with 3D Jones vectors and matrices.

Two-ports network, in 3D.

$$\begin{pmatrix} \begin{pmatrix} b_{1,x} \\ b_{1,y} \\ b_{1,z} \end{pmatrix} \\ \begin{pmatrix} b_{2,x} \\ b_{2,y} \\ b_{2,z} \end{pmatrix} \end{pmatrix} = S \begin{pmatrix} \begin{pmatrix} a_{1,x} \\ a_{1,y} \\ a_{1,z} \end{pmatrix} \\ \begin{pmatrix} a_{2,x} \\ a_{2,y} \\ a_{2,z} \end{pmatrix} \end{pmatrix}$$

$$S = \begin{pmatrix} \begin{pmatrix} S_{1,1,x,x} & S_{1,1,x,y} & S_{1,1,x,z} \\ S_{1,1,y,x} & S_{1,1,y,y} & S_{1,1,y,z} \\ S_{1,1,z,x} & S_{1,1,z,y} & S_{1,1,z,z} \end{pmatrix} & \begin{pmatrix} S_{1,2,x,x} & S_{1,2,x,y} & S_{1,2,x,z} \\ S_{1,2,y,x} & S_{1,2,y,y} & S_{1,2,y,z} \\ S_{1,2,z,x} & S_{1,2,z,y} & S_{1,2,z,z} \end{pmatrix} \\ \begin{pmatrix} S_{2,1,x,x} & S_{2,1,x,y} & S_{2,1,x,z} \\ S_{2,1,y,x} & S_{2,1,y,y} & S_{2,1,y,z} \\ S_{2,1,z,x} & S_{2,1,z,y} & S_{2,1,z,z} \end{pmatrix} & \begin{pmatrix} S_{2,2,x,x} & S_{2,2,x,y} & S_{2,2,x,z} \\ S_{2,2,y,x} & S_{2,2,y,y} & S_{2,2,y,z} \\ S_{2,2,z,x} & S_{2,2,z,y} & S_{2,2,z,z} \end{pmatrix} \end{pmatrix}$$

Scattering matrix of 3D Jones matrices.

One single entity to manage

- amplitude,
- phase,
- polarization,
- ports,
- orientation in space.

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Cascading networks.

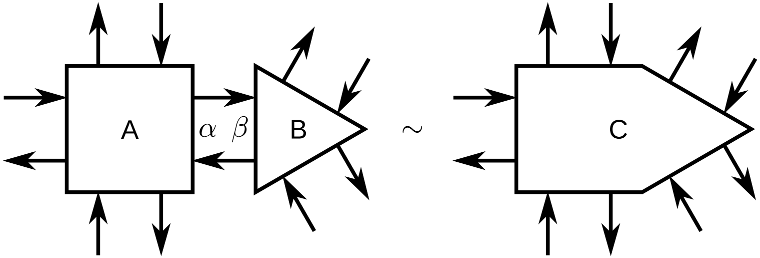


Figure: Two networks A and B connected by one port form a new network C.

Challenge.

Compute the scattering matrix of C.

Cascading networks.

After re-numbering the indices:

$$\forall (i,j) \in (\llbracket 1, n_A + n_B \rrbracket - \{\alpha, \beta\})^2$$

$$C_{i,j} = \begin{cases} A_{i,\alpha} R_A B_{\beta,\beta} A_{\alpha,j} + A_{i,j} & \text{if } i \leq n_a \text{ and } j \leq n_a \\ A_{i,\alpha} R_A B_{\beta,j} & \text{if } i \leq n_a \text{ and } j > n_a \\ B_{i,\beta} R_B A_{\alpha,j} & \text{if } i > n_a \text{ and } j \leq n_a \\ B_{i,\beta} R_B A_{\alpha,\alpha} B_{\beta,j} + B_{i,j} & \text{if } i > n_a \text{ and } j > n_a \end{cases}$$

$$R_A = (I - B_{\beta,\beta} A_{\alpha,\alpha})^{-1}$$

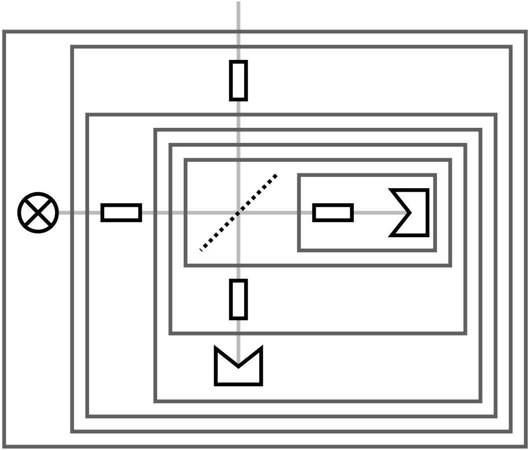
$$R_B = (I - A_{\alpha,\alpha} B_{\beta,\beta})^{-1}$$

This defines the scattering matrix of the equivalent network to A and B connected by one port.

Therefore.

By recursion, we can find the equivalent network of any system.

$$\boldsymbol{b} = S_{\text{HIFI-focal-plane-unit}}\,\boldsymbol{a}$$



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Ellipsis.

Explanations I am skipping here:

- How I compute the direction of propagation of each beam.
- How I build the scattering matrices of the different networks.

Kuddos to Martin Houde for his work on grids ♡.

> *Houde, Martin, et al. "Polarizing grids, their assemblies, and beams of radiation." Publications of the Astronomical Society of the Pacific 113.783 (2001): 622-638.*

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Conclusion.

Algorithm.

Given one tree of networks and loads.

1. For each set of parameter (frequency, diplexer offset, etc.)
   1. Set the parameters of each network.
   2. Propagate the direction of propagation.
   3. Summarize the whole system into one single network.
   4. Solve the fields on the ports of that loaded network.
2. Make plots.

Implementation.

Python using numpy.

Most of time spent in summarizing the networks.

Code parallelized to use all the CPU cores.

New algorithm available soon.

Faster, more stable, more features.

Part III

Parametric study.

Outline.

9 Introduction.

10 The entry grid.

11 One diplexer.

12 Two diplexers.

13 Next steps.

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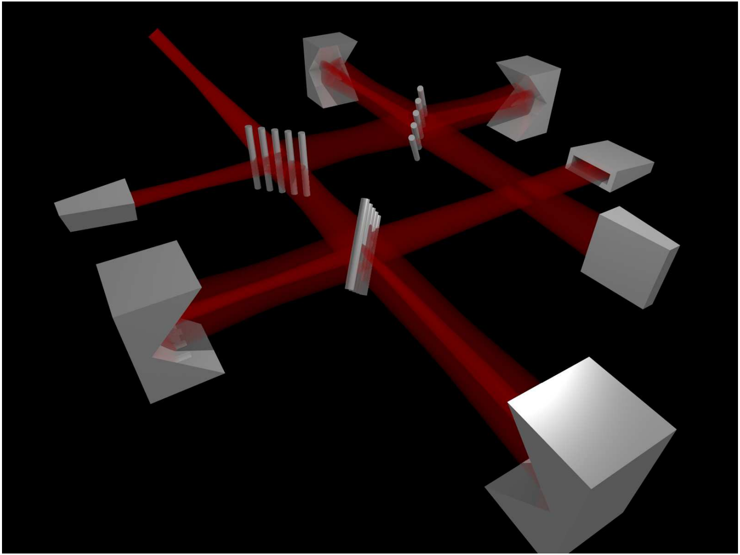
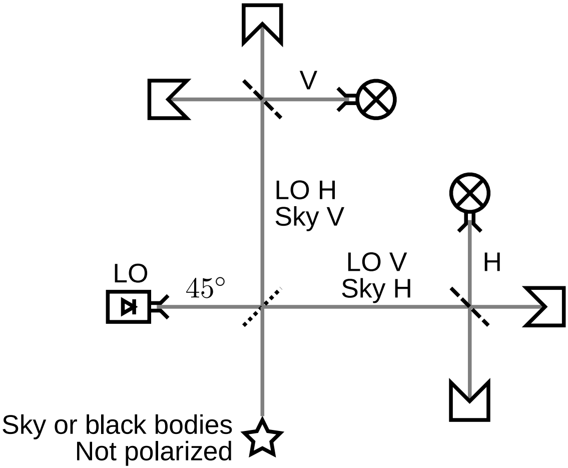
The setup.



The setup.



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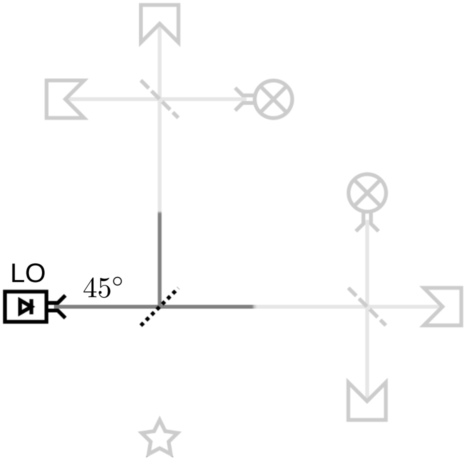

A very good grid.

Our entry grid is tilted to reflect H and transmit V.

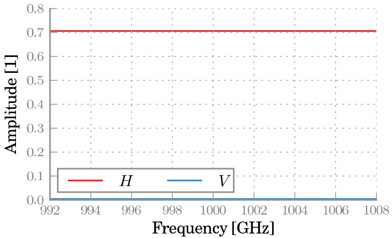

Figure: Reflection.

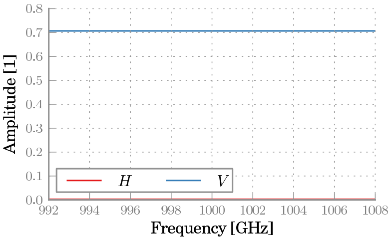

Figure: Transmission.

Matches expectations.

LO and sky at 45°, so $\sqrt{2}/2$ is reflected or transmitted.

A realistic grid.

Wire diameter $10\,\mu\mathrm{m}$, period $20\,\mu\mathrm{m}$, conductivity $3.5 \times 10^7\,\mathrm{S\,m^{-1}}$ (aluminum).

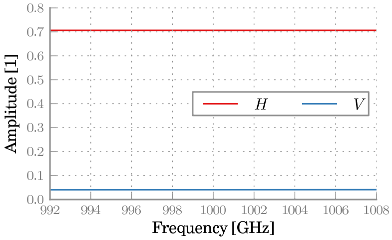

Figure: Reflection.

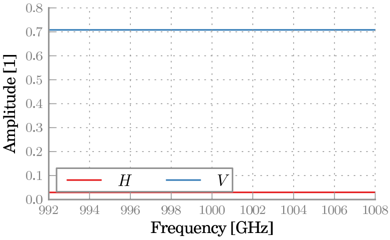

Figure: Transmission.

The grid leaks.

The rejected polarization is not at 0 anymore, the grid leaks.

Zooming in: slope.

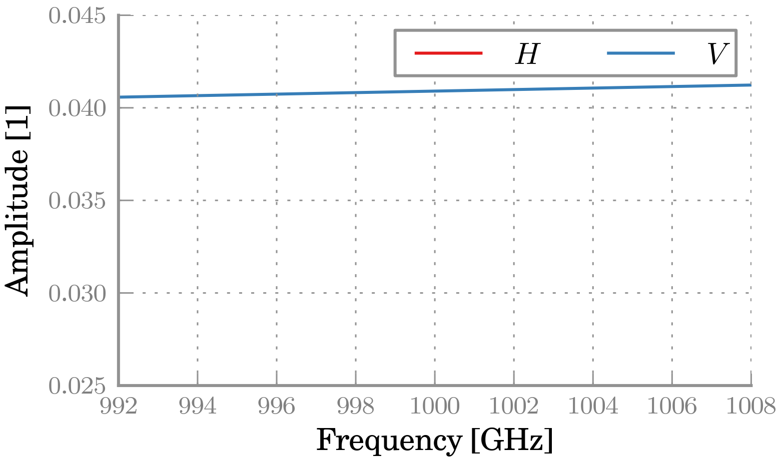

Figure: Reflection.

Zooming in: slope.

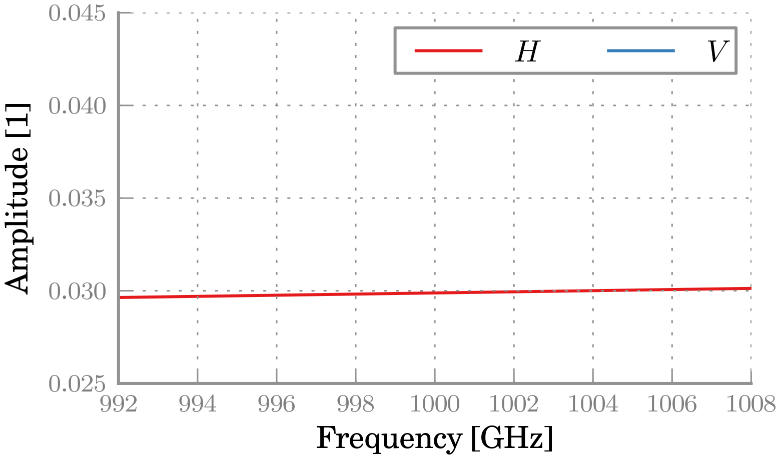

Figure: Transmission.

Realistic grid.

Summary.

Effect of the wire spacing and diameter.

- The leakage is not negligible.
- The grid leaks more in reflection: one diplexer will see more wrong-pol than the other.
- The grid leaks more at higher frequencies, but that effect is probably negligible. We do not have to ignore it though, our model gives it for free.

Realistic grid.

The missing bits.

We should also model

- the dielectric substrate (hard!),
- the metallic residuals on the dielectric substrate (easy).

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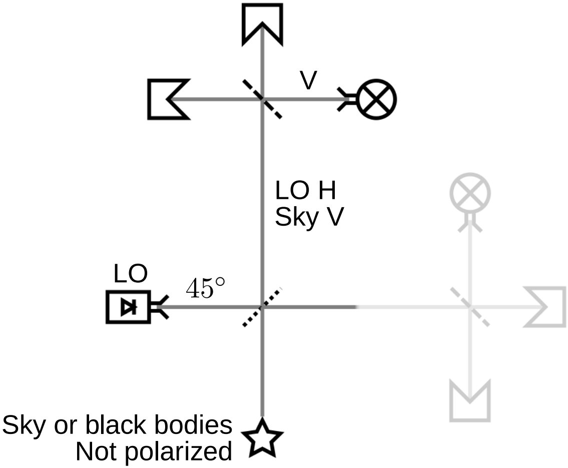
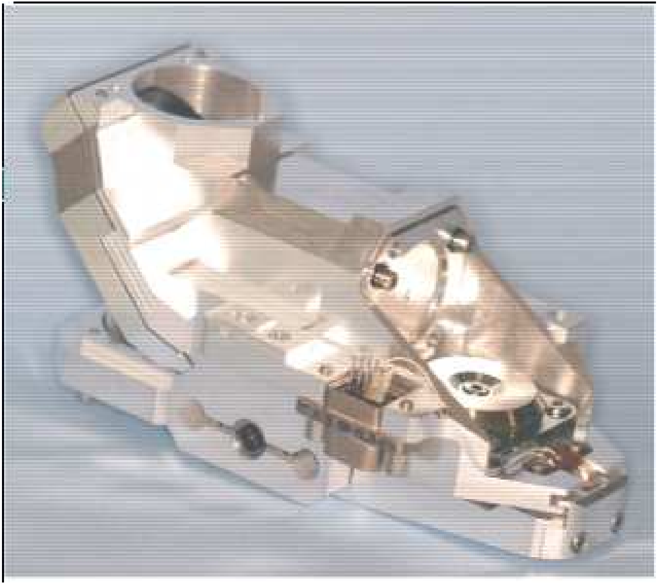


Photograph of a diplexer unit.



Ideal case.

Seen by the horn of the V mixer.

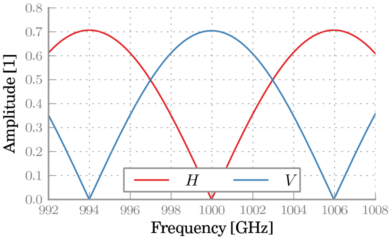


Figure: LO coupling.

V max at LO freq.

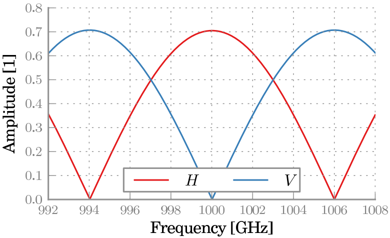


Figure: Sky coupling.

V max at LSB and USB freq.

Perfect tuning.

The theoretical diplexer offset pumps the mixer perfectly.

Ideal case.

LSB and USB folded on top of each other. There is no mixer model involved, it is a very naive fold that completely ignores the LO.

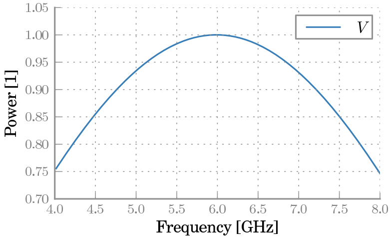


Figure: Folded sky.

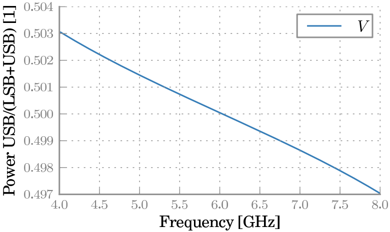


Figure: Sideband ratio.

With the realistic entry grid.

Slightly shifted left or right.

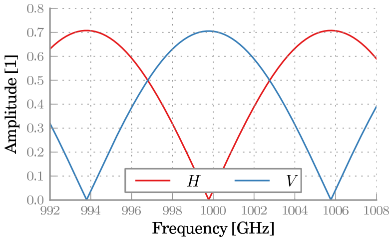


Figure: LO coupling.

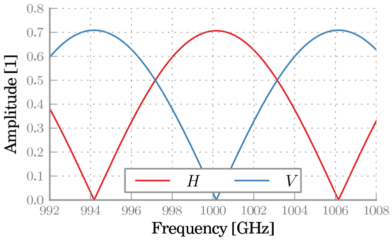


Figure: Sky coupling.

Imperfect tuning.

The theoretical diplexer offset is not enough. $\Longrightarrow$ Calibration.

With the realistic entry grid.

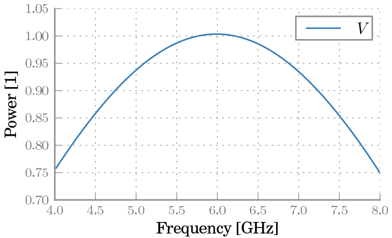


Figure: Folded sky.

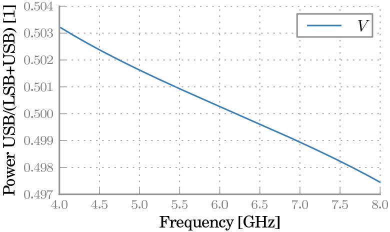


Figure: Sideband ratio.

Imperfect tuning.

The sideband ratio has shifted: .5 not at 6 GHz anymore.

Making the mixer reflective.

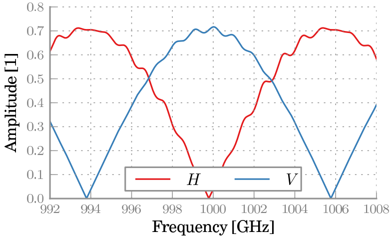


Figure: LO coupling.

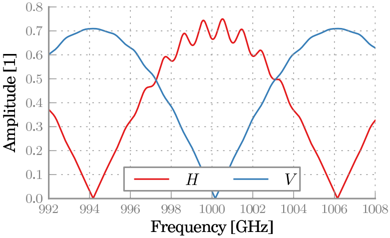


Figure: Sky coupling.

Where does the SW come from?

How can we have standing waves with only ONE reflective surface?

Blaming the diplexer grid, part 1: making it better.

Thinner and tighter grid.

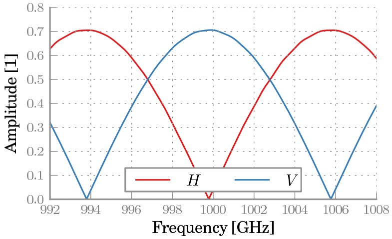


Figure: LO coupling.

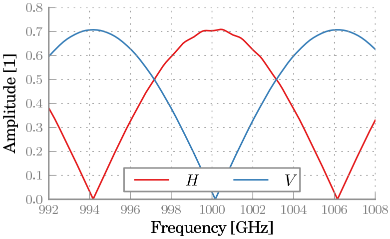
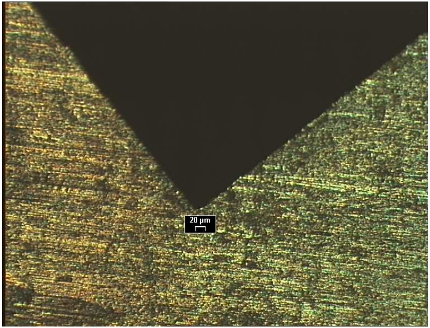


Figure: Sky coupling.

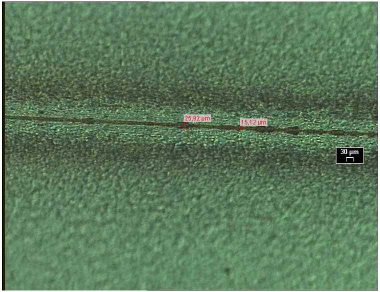
The diplexer grid is responsible.

The grid leaks, allowing the mixer to see itself in the rooftop mirrors.

Imperfect rooftop mirrors.



Imperfect junction of the two planes.



Imperfect rooftop mirrors.

Mixer reflects H only.

Rooftop mirror rotates only $95\,\%$.

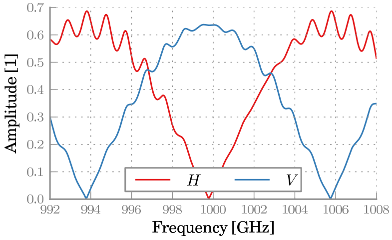


Figure: LO coupling.

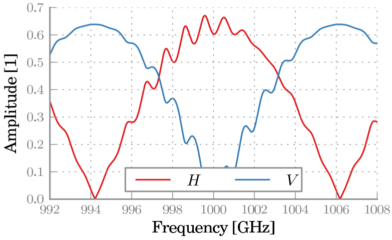


Figure: Sky coupling.

Our first cross-pol contamination!

SW in H and V despite the mixer reflecting only H.

Imperfect rooftop mirrors.

Mixer reflects H and V (standard behavior).
Rooftop mirror rotates only $95\,\%$.

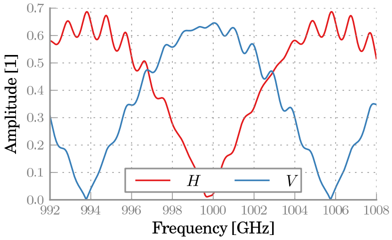


Figure: LO coupling.

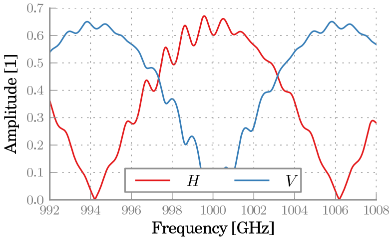


Figure: Sky coupling.

SW modulates SW.

Beat due to the different length of the diplexer arms.

Imperfect rooftop mirrors.

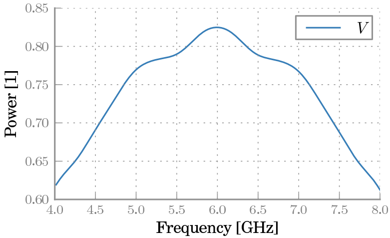


Figure: Folded sky.

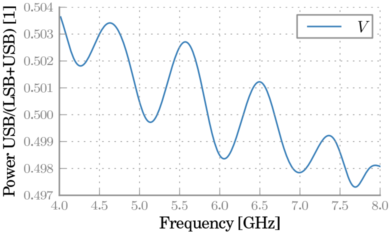


Figure: Sideband ratio.

Turning on the LO reflection.

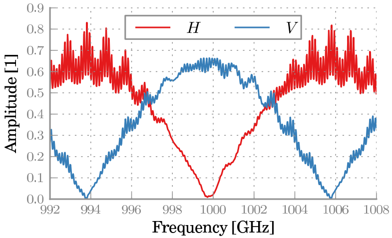


Figure: LO coupling.

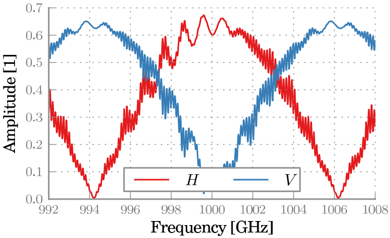


Figure: Sky coupling.

Fast LO-mixer standing wave.

This is why your spectra are ugly.

Turning on the LO reflection.

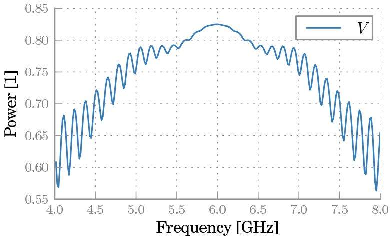


Figure: Folded sky.

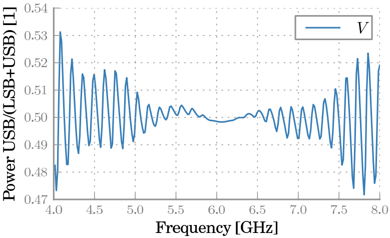


Figure: Sideband ratio.

Turning on the LO reflection.

Now, looking at the port that would go to the H mixer.

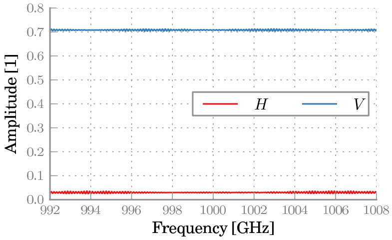


Figure: LO coupling.

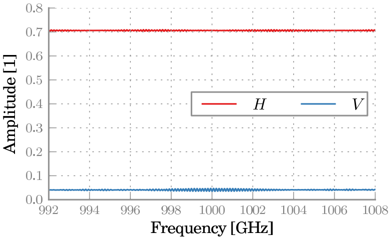
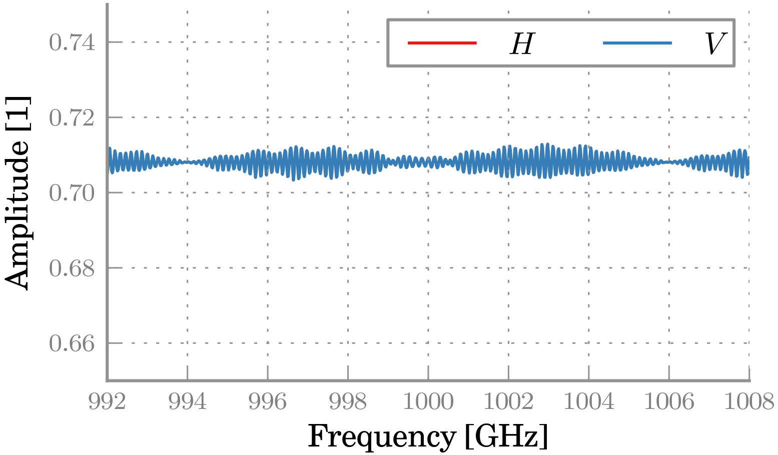


Figure: Sky coupling.



Cross talk.

The LO reflects the SW of the V mixer towards the H mixer.

Outline

9 Introduction.

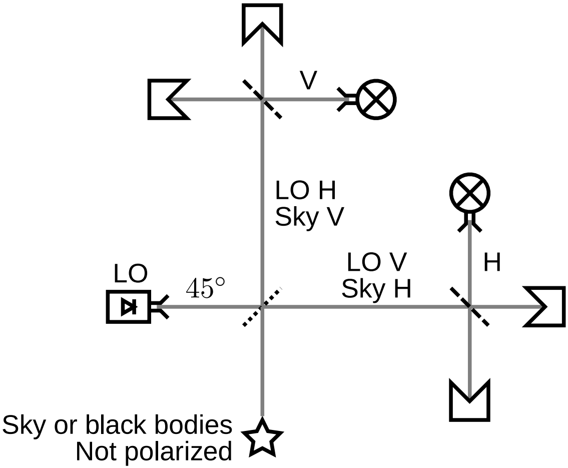
10 The entry grid.

11 One diplexer.

12 Two diplexers.

13 Next steps.

The setup



Effect of tuning on the cross talk.

Well-tuned diplexers.

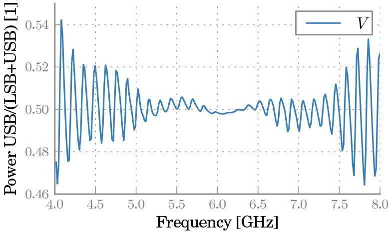


Figure: V sideband ratio.

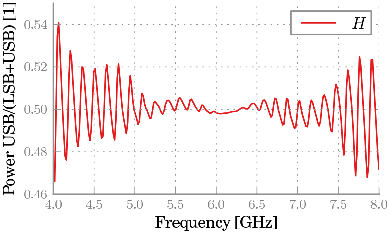


Figure: H sideband ratio.

Effect of tuning on the cross talk.

H diplexer well tuned, V diplexer badly tuned.

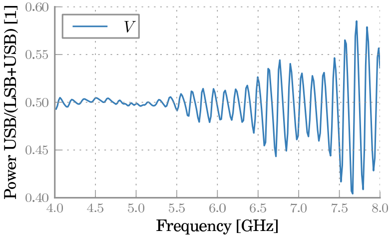


Figure: V sideband ratio.

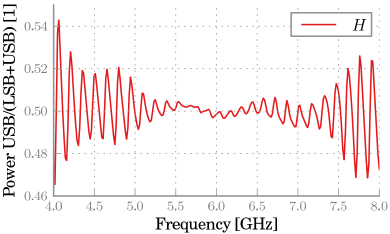
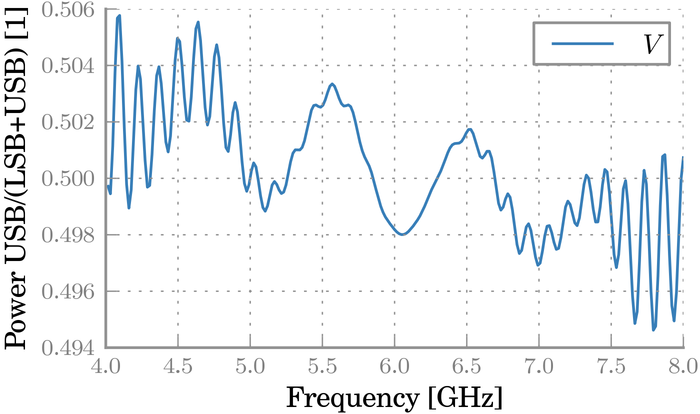


Figure: H sideband ratio.

Crosstalk.

Detuning one diplexer changes the SBR on the other.

With a 10 dB attenuator in the LO path.



With attenuator.

The main SW is the mixer-rooftop one due to the leaky grids.

Outline

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Next steps.

From qualitative to quantitative.

- Gather lengths, conductivities and other HIFI parameters.
- Build a a mixer-model (Tucker. 3-ports?)
- Grids' substrate.
- Will gaussian beams matter?